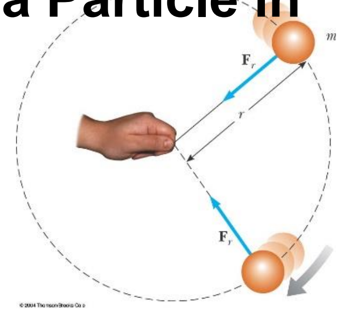
Lecture 06 Circular Motion and Other Applications of Newton's Laws

6.1 Newton's Second Law for a Particle in Uniform Circular Motion



$$\vec{a}_r = -\frac{v^2}{r}\hat{r}\ ,\quad \sum \vec{F}_i = -m\frac{v^2}{r}\cdot\hat{r}$$

We call them centripetal acceleration and centripetal force because they are directed toward the center of the circle.

Example: A car travels on a circular roadway of radius r. The roadway is flat. The car travels at a high speed v, such that the friction force causing the centripetal acceleration is the maximum possible value. If the same car is now driven on another flat circular roadway of radius 2r, and the coefficient of friction between the tires and the roadway is the same as on the first roadway, what is the maximum speed of the car such that it does not slide off the roadway?

$$m\frac{v^2}{r}\cdot\hat{r} - mg\cdot f_k\cdot\hat{r} = 0\ ,\quad m\frac{v'^2}{2r}\cdot\hat{r} - mg\cdot f_k\cdot\hat{r} = 0$$

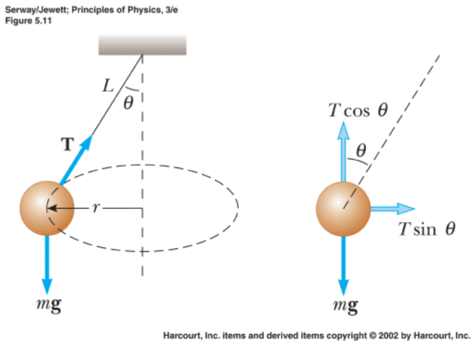
Example: How fast can it spin?

An object of mass 0.500 kg is attached to the end of a cord whose length is 1.50 m. The object is whirled in a horizontal circle. If the cord can withstand a maximum tension of 50 N, what is the maximum speed the object can have before the cord breaks?

$$\vec{T} = -T\cdot\hat{r}\ ,\quad -T\cdot\hat{r} + m\frac{v^2}{r}\cdot\hat{r} = 0\ ,\quad v = \sqrt{\frac{rT}{m}} = \sqrt{\frac{1.5\cdot 50}{0.5}} = 12.2$$

Example: The Conical Pendulum

A small object of mass m is suspended from a string of length L. The object revolves in a horizontal circle of radius r with constant speed v. Find (a) the speed of the object, and (b) the period of revolution

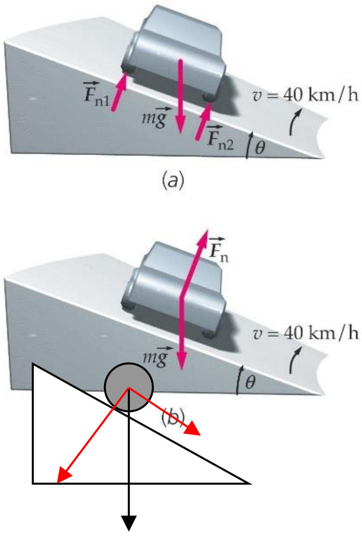


$$-T\sin\theta\cdot\hat{r} + T\cos\theta\cdot\hat{z} - mg\cdot\hat{z} = -m\frac{v^2}{r}\cdot\hat{r}$$

$$T = \frac{mg}{\cos\theta}, \quad v = \sqrt{gr\tan\theta} = \sqrt{gL\sin\theta\tan\theta}, \quad time = \frac{2\pi r}{v} = \frac{2\pi L\sin\theta}{v} =$$

Banked Curves

See what happens if a particle moves with varying speed in a circular path.

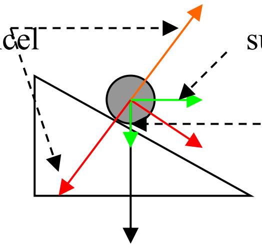


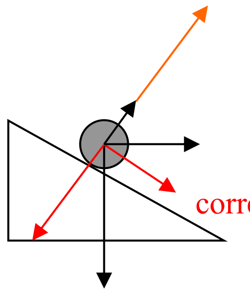
Example: A curve of radius 30 m is banked at an angle θ. Find θ for which a car can round the curve at 40 km/h even if the road is covered with ice that friction is negligible.

$mg\sin\theta$ along the road surface must be canceled by the required force of circular motion:

$$ma\cos\theta = m\frac{v^2}{r}\cos\theta.$$

$$\rightarrow \quad mg\sin\theta = m\frac{v^2}{r}\cos\theta$$



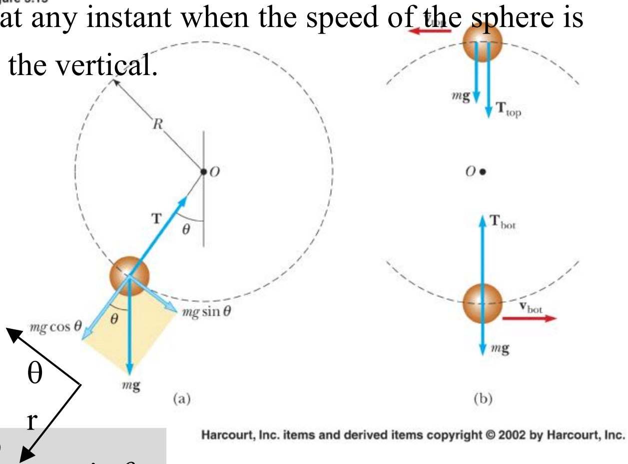


6.2 Nonuniform Circular Motion

Example: Follow the Rotating Ball

A small sphere of mass m is attached to the end of a cord of length R which rotates under the influence of the gravitational force in a vertical circle about a fixed point O. Let us determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle theta with the vertical.

Serway/Jewett; Principles of Physics, 3/e
Figure 5.15



$$\vec{r} = -R\sin\theta\cdot\hat{i} - R\cos\theta\cdot\hat{j},$$

$$-T\cdot\hat{r} + mg\cos\theta\cdot\hat{r} = -m\frac{v^2}{R}\cdot\hat{r}$$

$$T = m\left(\frac{v^2}{R} + g\cos\theta\right)$$

To find the eq. of motion:

$$F = m\frac{dv}{dt} = mR\frac{d^2\theta}{dt^2} = -mg\sin\theta, \quad R\frac{d^2\theta}{dt^2} = -g\sin\theta,$$

$$R\frac{d^2\theta}{dt^2}\frac{d\theta}{dt}=-g\sin\theta\frac{d\theta}{dt},\quad \frac{d}{dt}\left(\frac{1}{2}\dot{\theta}^2\right)=-\frac{g}{R}\sin\theta\frac{d\theta}{dt},$$

$$d\left(\frac{1}{2}\dot{\theta}^2\right)=-\frac{g}{R}\sin\theta d\theta \;\text{->}\; \frac{1}{2}\dot{\theta}^2=\frac{g}{R}\cos\theta+C\text{, if T = 0 on top ->}\; v_{top}=R\dot{\theta}_{top}=\sqrt{Rg}$$

$$\frac{1}{2}\dot{\theta}^2=\frac{g}{R}\cos\theta+\frac{3}{2}g,\quad \frac{d\theta}{dt}=\sqrt{2\frac{g}{R}\cos\theta+3\frac{g}{R}},\quad v(\theta)=R\dot{\theta}=\sqrt{2Rg\cos\theta+3Rg}$$

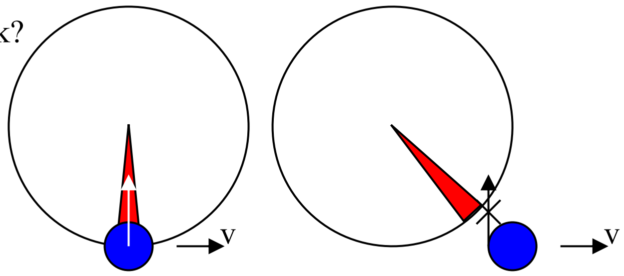
6.3 Motion in Accelerated Frames

Is it in an inertia frame?

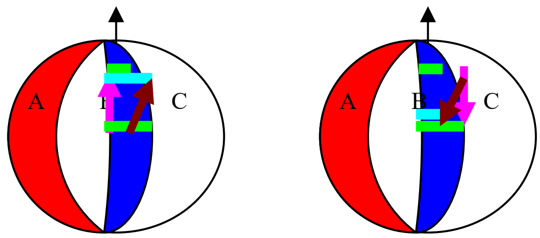
<u>In car</u>: if the friction force is not sufficiently great to allow somebody sitting in car to travel along the circular path, he may experience a ghost force

If he still move with the car, he may experience centrifugal force. If he does not move with the car, he may be throw away in ?? direction.

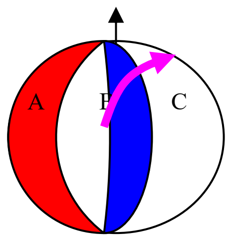
How does the drying machine work?



<u>On the Earth</u>: Coriolis force??

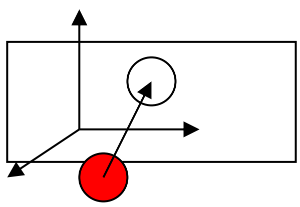


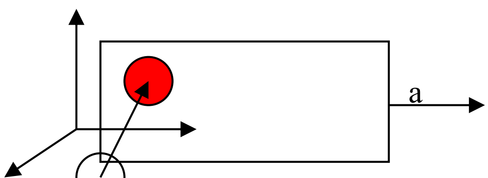
counterclockwise in the northern hemisphere and clockwise in the southern hemisphere

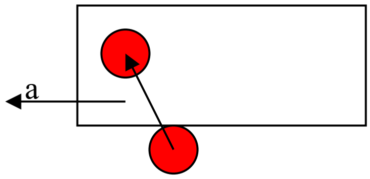


In the frame of the Earth, you believe that a force exists to drag the body.

<u>In an accelerated train:</u>







6.4 Motion in the Presence of Resistive Force

Model 1: Resistive force proportional to objective velocity

$$\vec{R} = -b\vec{v}\,,\quad F = mg - bv = m\frac{dv}{dt}\,,\quad m\frac{dv}{mg-bv} = dt\,,\quad d[-\frac{m}{b}\ln(mg-bv)] = dt$$

$$\int_0^t dt = \int_0^v d[-\frac{m}{b}\ln(mg-bv)]\,,\quad t = -\frac{m}{b}\ln\frac{mg-bv}{mg}\,,\quad \frac{mg-bv}{mg} = \exp(-\frac{b}{m}t)$$

$$v = \frac{mg}{b}(1-\exp(-\frac{b}{m}t))$$

$$v = v_t \Leftarrow F = 0\,,\quad v_t = \frac{mg}{b}\,,\quad \tau = \frac{m}{b}$$

$$v = v_t(1-\exp(-\frac{t}{\tau}))$$

time constant: $\tau = \frac{m}{b}\,,\quad 1-e^{-1} = 0.632 = 63.2\%$

Example: A sphere falling in oil

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil. The sphere approaches a terminal speed of 5.00 cm/s. Determine (a) the time constant τ and (b) the time it takes the sphere to reach 90% of its terminal speed.

(a) $v_t = \frac{mg}{b} = 0.05\cdot m/s\,,\quad b = \frac{0.002\cdot 9.8}{0.05} = 0.392\cdot\frac{kg}{s}\,,\quad \tau = \frac{m}{b} = \frac{0.002}{0.392} = 5.1\cdot 10^{-3}$

(b) $t = -5.1\cdot 10^{-3}\ln\frac{mg - b\cdot 0.9\frac{mg}{b}}{mg} = -5.1\cdot 10^{-3}\ln(0.1) = 11.7\cdot 10^{-3}$

Model 2: Resistive force proportional to object speed square – Air Drag at High Speeds

$$R = -\frac{1}{2}D\rho Av^2\,,\quad F = mg - \frac{1}{2}D\rho Av^2\,,\quad F = 0 \Rightarrow v_t = \sqrt{\frac{2mg}{D\rho A}}$$

for a human body in free fall motion with a square of velocity dependent drag force, the terminal speed is: $v_t \approx \sqrt{\dfrac{2*60*9.8}{0.6*\dfrac{0.028}{0.0224}*1}} = 39.6\dfrac{m}{s} = 143 \cdot {}^{km}\!/_{hr}$

Object	Mass (kg)	Cross-Section (m^2)	V_t (m/s)
Sky diver	75	0.70	60
Bassball	0.145	4.2×10^{-3}	43
Golfball	0.046	1.4×10^{-3}	44
Hailstone	4.8×10^{-4}	7.9×10^{-5}	14
Raindrop	3.4×10^{-5}	1.3×10^{-5}	9.0

Example: If a falling cat reaches a first terminal speed of 97 km/h while it is tucked in and then stretches out, doubling *A*, how fast is it falling when it reaches a new terminal speed?

$$\frac{v_1}{v_2} = \sqrt{\frac{A_2}{A_1}}, \quad \frac{97}{v_2} = \sqrt{\frac{2A}{A}}, \quad v_2 = \frac{97}{\sqrt{2}} = 68.6\,{}^{m}\!/_{s}$$

Example: A raindrop with radius R = 1.5 mm falls from a cloud that is at height h = 1200 m above the ground. The drag coefficient D for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water ρ_w is 1000 kg/m^3, and the density of air ρ_a is 1.2 kg/m^3.

(a) What is the terminal speed of the drop?

$$A = \pi R^2 = 3.14 \cdot (0.0015)^2 = 7 \cdot 10^{-6} m^2, \quad m = 1000 \cdot \frac{4}{3}\pi \cdot (0.0015)^3 = 1.4 \cdot 10^{-5}$$

$$v_t \approx \sqrt{\frac{2mg}{D\rho A}} = \sqrt{\frac{2 \cdot 1.4 \cdot 10^{-5} \cdot 9.8}{0.6 \cdot 1.2 \cdot 7 \cdot 10^{-6}}} = 7.4 \cdot m/s = 27 \cdot km/hr$$

Numerical Integration: Euler's Method

If you cannot solve the exact solutions of x(t), you need to express it numerically. In the real world you may always need the numerical representations of motions.

EX1: Consider the initial value problem $y' = 0.1\sqrt{y} + 0.4x^2$, $y(2) = 4$. Use Euler's method to obtain an approximation of $y(2.5)$ using $h = 0.1$ and $h = 0.05$.

x	y	y'	
0	2	4	1.8
1	2.1	4.18	1.96845
2	2.2	4.376845	2.145209
3	2.3	4.591366	2.330275
4	2.4	4.824393	2.523645
5	2.5	5.076758	2.725317

x	y	y'	
0	2	4	1.8
1	2.05	4.09	1.883237
2	2.1	4.184162	1.968552
3	2.15	4.282589	2.055944
4	2.2	4.385387	2.145413
5	2.25	4.492657	2.236959
6	2.3	4.604505	2.330581
7	2.35	4.721034	2.426279
8	2.4	4.842348	2.524053
9	2.45	4.968551	2.623902
10	2.5	5.099746	2.725826

given an acceleration $a(x, v, t)$, if we can not find the velocity & position by integration and differentiation, you may express it by the Euler method:

$$a \approx \frac{\Delta v}{\Delta t} \Rightarrow v(t + \Delta t) = v(t) + a\Delta t \,, \quad v(t) \approx \frac{\Delta x}{\Delta t} \Rightarrow x(t + \Delta t) = x(t) + v(t) \cdot \Delta t$$

a = 2 m/s^2, Δt = 0.1 s, v(0) = 0, x(0) = 0 -					v(t) = at, x(t)=1/2*at^2	
Step	t	v	x		v(t)	x(t)
0	0.	0	0		0	0
1	0.1	0.2	0		0.2	0.01
2	0.2	0.4	0.02		0.4	0.04
3	0.3	0.6	0.06		0.6	0.09
4	0.4	0.8	0.12		0.8	0.16