Lecture 08 Conservation of Energy

8.1 The Nonisolated System: Conservation of Energy

work, mechanical waves, heat (driving by a temperature difference), matter transfer (convection), electrical transmission (by means of electric currents), electromagnetic radiation (photons)

Conservation of Energy Equation:

$$\Delta K + \Delta U + \Delta E_{\text{int}} = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$$

What is the energy of a visible photon? (eV = ? Joule)

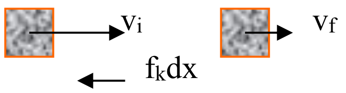
We can neither create nor destroy energy – energy is always conserved.

work-kinetic energy valid only when the object can be modeled as a particle

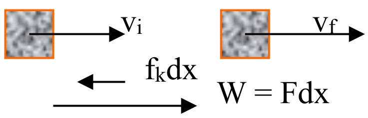
if a frictional force exists, how to express the work with it?

What's the difference between kinetic friction and resistive force?

If no external work applied to the system, the loss of kinetic energy will transfer to work done by the kinetic friction force. $\frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2} = \Delta K = -f_k \Delta x$



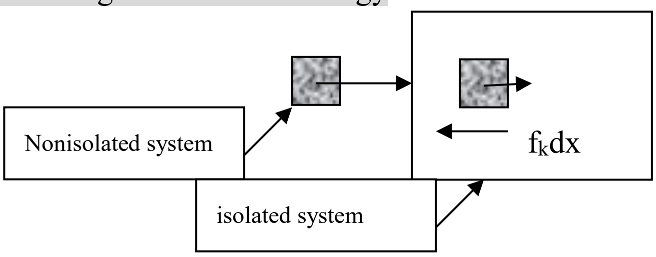
If external force exist, the system may gain energy from the external work and loss energy to the kinetic friction force. $\Delta K = W - f_k \Delta x$



We said in previous section that no energy can be created or destroyed. Where does the work done by the kinetic friction force go? -> Internal Energy

$$\Delta E_{system} = \Delta K + \Delta E_{\text{int}} = 0$$

$$\Delta E_{\text{int}} = f_k d$$



The result of a friction force is to transfer kinetic energy into internal energy.

Frictional force -> Resistive force -> Drag force??

Example: A car traveling at a speed v slides a distance d to a halt after its brake lock. Assuming that the car's initial speed is instead 2v at the moment the brakes lock, estimate the distance it slides.

4d

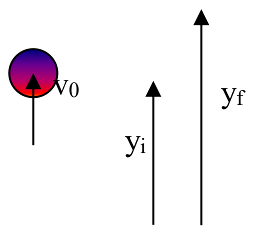
Example: A block of mass 1.6 kg is attached to a horizontal sprint that has a force constant of 1.0 X 10^3 N/m. The spring is compressed 2.0 cm and is then released from rest. Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4.0 N retards its motion from the moment it is released.

$$W = V = \frac{1}{2}kx^2 = f_k d + \frac{1}{2}mv^2$$

8.2 The Isolated System

the work done by the gravitational force:

$$W = mg(-\hat{z})\cdot(y_f - y_i)\hat{z} = mgy_i - mgy_f, \quad y_f < y_i$$



transform to mechanical energy of the object

$$W = \Delta K = mgy_i - mgy_f$$

notice the minus sign

$$W = -\Delta U_g$$

work is only **intermediate** substitutable quantity to express the energy transfer

$W = \Delta K = -\Delta U_g$ -> $\Delta K + \Delta U_g = 0$ kinetic energy and potential energy of an object in the system

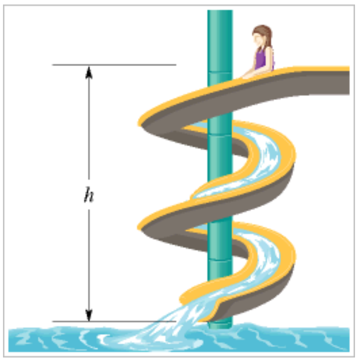
Define Mechanical Energy: E_{mech}, $E_{mech} = K + U_g$

If $\Delta K + \Delta U_g = 0$ -> $K_f - K_i + U_{gf} - U_{gi} = 0$ -> $K_i + U_{gi} = K_f + U_{gf}$

-> $\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$ -> conservation of mechanical energy for an isolated system

This result is called the **principle of conservation of mechanical energy.** (Now you can see where *conservative* forces got their name.) We can write this principle in one more form, as $\Delta E_{mech} = \Delta K + \Delta U_g = 0$

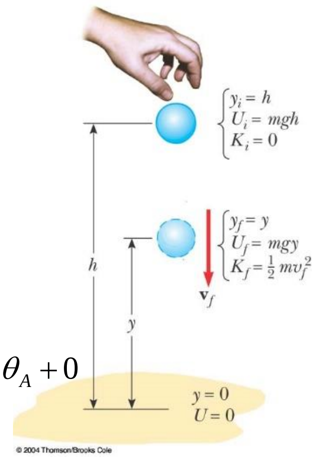
Example: A child of mass m is released from rest at the top of a water slide, at height h = 8.5 m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



$$v = \sqrt{2gh} = \sqrt{2*9.8*8.5} = 13\,\text{m/s}$$

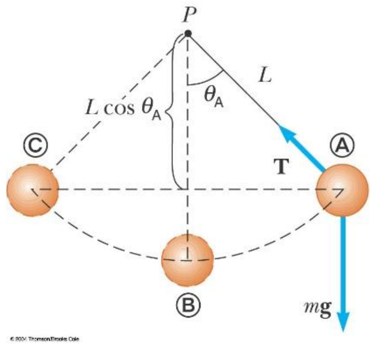
Example: Ball in Free Fall

$$mg(h-y) = \frac{1}{2} m v_f^2$$



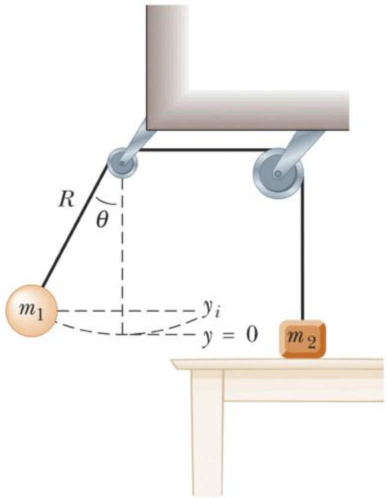
Example: The Pendulum

$$-mgL + \frac{1}{2} m v_B^2 = -mgL\cos\theta_A + 0$$



Serway/Jewett; Principles of Physics, 3/e
Figure 7.6

Example: Two blocks are connected by a massless cord that passes over two frictionless pulleys, as in Figure. One end of the cord is attached to an object of mass m_1=3.00 kg that is a distance R=1.2m from the pulley on the left. The other end of the cord is connected to a block of mass m_2=6kg resting on a table. From what angle must the 3.00 kg mass be released in order to just lift the 6.00 kg block off the table?

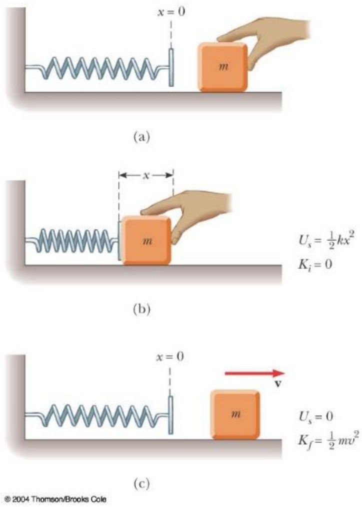


Energy conservation: $m_1 g h = \frac{1}{2} m_1 v^2$,

$gR(1-\cos\theta) = \frac{1}{2} v^2$,

Force balance: $m_2 g = m_1 g + m_1 \frac{v^2}{R}$,

$$m_2 g = m_1 g + m_1 \cdot 2g(1-\cos\theta), \quad \cos\theta = \frac{3m_1 - m_2}{2m_1}$$



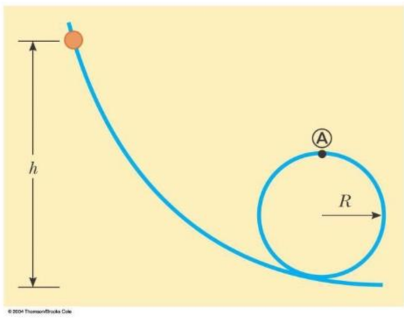
Elastic Potential Energy

$$F = -kx, \quad U = W_{Fapp} = \int (kx)dx = \frac{1}{2}kx_f^{\ 2} - \frac{1}{2}kx_i^{\ 2}$$

$E_{mech} = U + K = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, if the energy is conserved in the isolated system, $\Delta E_{mech} = \Delta U + \Delta K = 0$ ->

$$\frac{1}{2}mv_i^{\ 2} + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^{\ 2} + \frac{1}{2}kx_f^2$$

Example: A bead slides without friction around a loop-the-loop. The bead is released from a height h = 3.50R. (a) What is its speed at point A? (b) How large is the normal force on it if its mass is 5.00 g? Hint: (a) Transfer potential energy to kinetic energy, (b) calculate the centripetal force that the wall exerted on the particle, the force that the particle exert on the wall may need to subtract the gravitational term



(a) $v = \sqrt{3gR}$

(b) $-2mg$ normal force is from the plane in downward direction

See AF0608 & AF0816.

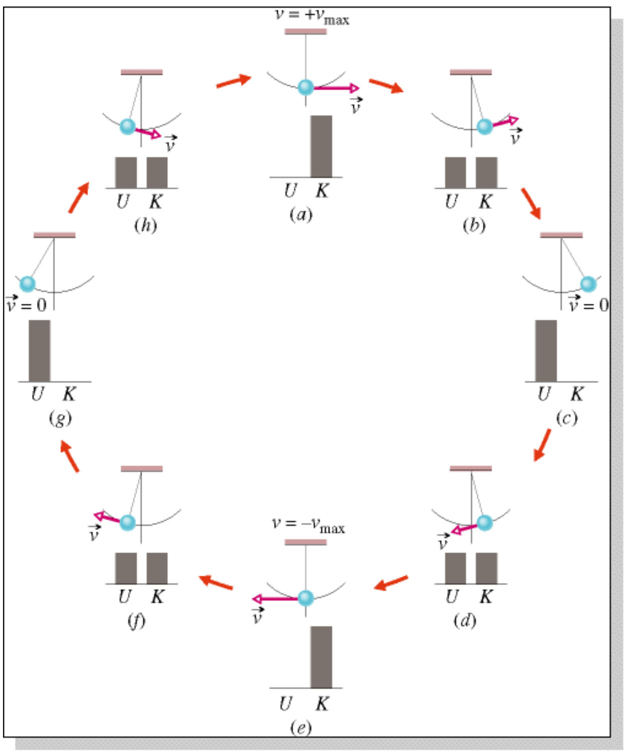
Gravitational potential energy:

$$W = \int_{xi}^{xf} F(x)dx = -\Delta U$$

$$\Delta U = -\int_{xi}^{xf} F(x)dx$$

$$\Delta U = -\int_{yi}^{yf} (-mg)dy = mg\int_{yi}^{yf} dy$$

$$\Delta U = mg[y]_{yi}^{yf} = mg(y_f - y_i) = mg\Delta y$$



Click on the image to start the simulation elastic potential energy:

$$\Delta U = -\int_{xi}^{xf} F(x)dx = -\int_{xi}^{xf} -kxdx = k\int_{xi}^{xf} xdx = \frac{1}{2}k\left[x^2\right]_{xi}^{xf}$$

$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

The Work-Energy Theorem (compared with the work-kinetic energy theorem)

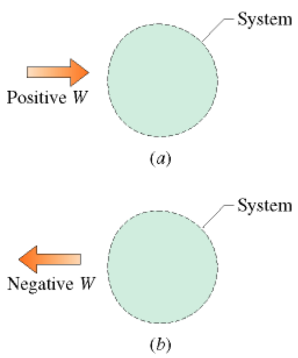
$W = \Delta E = \Delta E_{mech} + \Delta E_{th} + \Delta E_{\text{int}}$

$\Delta E_{mech} = \Delta K + \Delta U$

isolated system -> $W = \Delta E = 0$

8.3 Situations Involving Kinetic Friction

8.4 Changes in Mechanical Energy for Nonconservative Forces



No Friction Involved:

$W = \Delta E_{mech} = \Delta K + \Delta U$

Friction Involved:

1. Consider the kinetic energy only: $\Delta K = -f_k d$
2. Consider the mechanical energy: $\Delta E = \Delta K + \Delta U_g = -f_k d$

Derived from the force law: $F - f_k = ma$, $v^2 = v_0^2 + 2ad$,

$$Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + f_k d$$

By doing experiment, we found that the block and the portion of the floor along which it slides become warmer as the block slides

$\Delta E_{th} = f_k d$

$W = Fd = \Delta E_k + \Delta E_{th}$

Example: A food shipper pushes a wood crate of cabbage heads (total mass m = 14 kg) across a concrete floor with a constant horizontal force $\vec{F}$ of magnitude 40 N. In a straight-line displacement of magnitude d = 0.50 m, the speed of the crate decreases from v_0 = 0.60 m/s to v = 0.20 m/s. (a) How much work is done by force $\vec{F}$, and on what system does it do the work?

$W = 40 \cdot 0.5 = 20 \cdot J$

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

$$\Delta E_{th} = W - \Delta K = 20 \cdot J - \left(\frac{1}{2}14 \cdot 0.2^2 - \frac{1}{2}14 \cdot 0.6^2\right) = 22.24 \cdot J$$

Example: A child of mass m rides on an irregularly curved slide of height h = 2.00 m. The child starts at rest on the top.

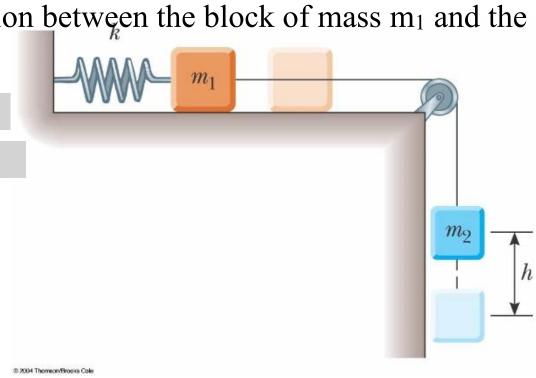
(a) Determine his speed at the bottom, assuming no friction is present.

$$v = \sqrt{2gh}$$

(b) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that v_f = 3.00 m/s and m = 20.0 kg.

$$\Delta E_{mech} = mgh - \frac{1}{2}mv^2 = mgh - \frac{1}{2}20.0 * 3.00^2 = 302J$$

Example: Two blocks are connected by a light string. The block of mass m_1 is connected to a spring of force constant k. The system is released from rest when the spring is unstreched. If the block of mass m_2 falls a distance of h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

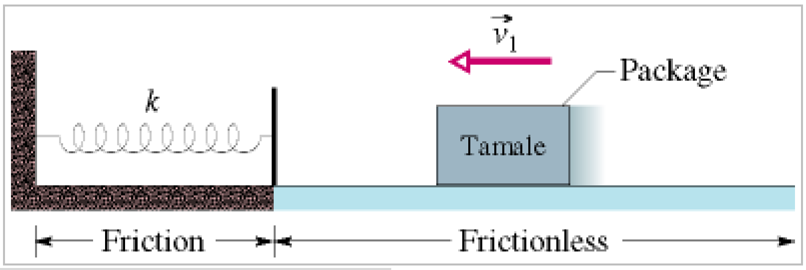


Hint: Potential energy is provided to stretch the spring and to the loss of kinetic friction.

Overdamped system?

$$\mu_k = \frac{m_2 gh - \frac{1}{2}kh^2}{m_1 gh}$$

Example: A 2.0 kg package of tamale slides along a floor with speed $v_1 = 4.0$ m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude 15 N, acts on it. The spring constant is 10,000 N/m. By what distance d is the spring compressed when the package stops?



$-5000 \cdot d^2 = 15 \cdot d - 16$, $5000d^2 + 15d - 16 = 0$,

$$d = \frac{-15 \pm \sqrt{15^2 + 4 \cdot 5000 \cdot 16}}{2 \cdot 5000} \approx 0.056$$

Systems With Chemical Energy

Example: You are driving a 1000-kg gasoline-powered car at a constant speed of 100 km/h (27.8 m/s) up a 10-percent grade. (a) If the efficiency is 15 percent, what is the rate at which the chemical energy of the car-earth-atmosphere system changes?

$\tan\theta = 10\% = 0.1$

$$\frac{\Delta E_{chem}}{\Delta t} * 0.15 = mg\frac{\Delta h}{\Delta t} = mgv\sin\theta \sim mgv\tan\theta$$

$$-\frac{dE_{chem}}{dt} = \frac{1000 * 9.8}{0.15} 27.8 * 0.1 = 182kW$$

Mass and Energy

In 1905, Albert Einstein published his special theory of relativity, a result of which is the famous equation $E_0 = mc^2$.

A particle or system of mass m has "rest" energy mc^2. The energy is intrinsic to the particle. Energy in atomic and nuclear physics are usually expressed in units of electron volts (eV). A convenient unit for the masses of atomic particles is eV/c^2.

Particle	Symbol	Rest Energy (MeV)
Electron	e^-	0.5110
Positron	e^+	0.5110
Proton	p	938.272
Neutron	n	939.565

Deuteron	d	1875.613
Triton	t	2808.410
Helium-4 (alpha particle)	^4He	3727.379

The increase in mass with an increase in energy of 4 J is

$$\Delta m = \frac{E}{c^2} = \frac{4}{\left(3\times 10^8\right)^2} = 4.44\times 10^{-17} \text{ kg.}$$

Nuclear Energy

Example: A typical nuclear fusion reaction is written $^2H+^3H \to ^4He+n$. How much energy is released in this fusion reaction?

$1875.613 + 2808.410 - 3727.379 - 939.565 \sim 17.1$ MeV

Newtonian Mechanics and Relativity

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mc^2\frac{v^2}{c^2} = \frac{E_0}{2}\frac{v^2}{c^2} \rightarrow \frac{v}{c} = \sqrt{\frac{2K}{E_0}} \quad E_0 : \text{rest mass energy}$$

Newtonian mechanics is valid of the speed of the particle is much less than the speed of light.

Quantization of Energy

Plank's constant: $h = 6.626\times 10^{-34}$ J·s

The quantized energy of an oscillator:

$$E = \left(n+\frac{1}{2}\right)\hbar\omega = \left(n+\frac{1}{2}\right)hf \rightarrow \text{ground state energy } E_0 = E_{n=0} = \left(0+\frac{1}{2}\right)hf = \frac{1}{2}hf$$

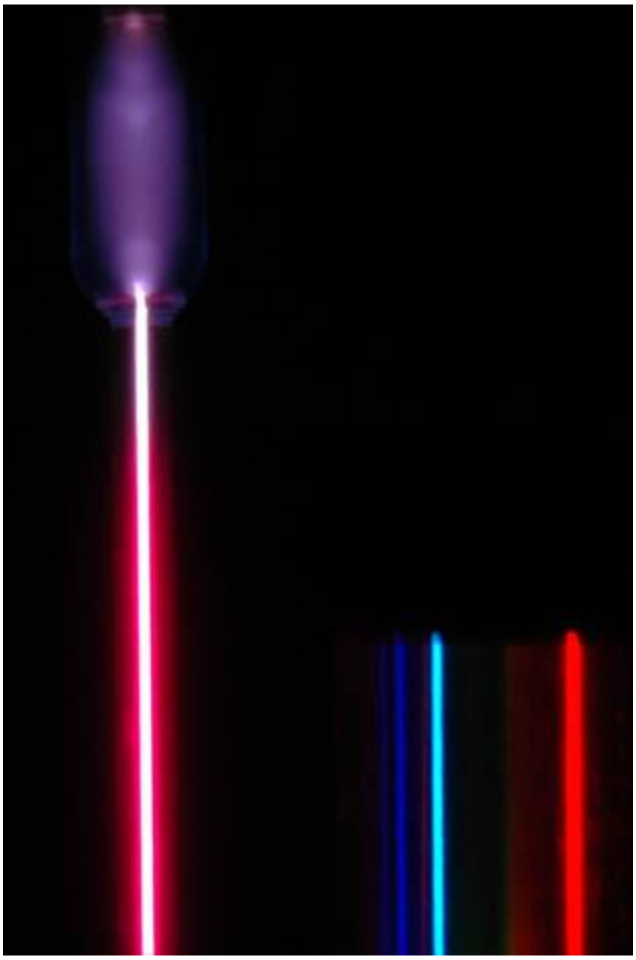
The quantized energy of a photon: $E_{photon} = hf$, f is the frequency of the electromagnetic radiation.

For macroscopic bound system, the oscillation frequencies for a spring system are about 1 to 10 Hz. If $f = 10$, the spacing between allowed levels is $hf \sim 6\times 10^{-33}$ J. The energy of a macroscopic system is in the order of 1 J. The energy level spacing is too small to be observable.

Example: For a diatomic molecule, a typical frequency of vibration is 10^{14}, and a typical energy of 10^{-19} J. The spacing between allowed levels is then

$$E_{n+1} - E_n = h\nu = 6\times 10^{-20} \text{ J}, \quad \Delta E = \frac{6\times 10^{-20}}{1.602\times 10^{-19}} \sim 0.4 \text{ eV}$$

Since the energy of molecule is in the order of 1 eV, the energy spacing is not negligible.



At left is a hydrogen spectral tube excited by a 5000 volt transformer. The three prominent hydrogen lines are shown at the right of the image through a 600 lines/mm diffraction grating.

An approximate classification of spectral colors:

Violet (380-435nm)
Blue(435-500 nm)
Cyan (500-520 nm)
Green (520-565 nm)
Yellow (565- 590 nm)
Orange (590-625 nm)
Red (625-740 nm)

8.5 Power

$$P = \frac{dE}{dt}$$

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F} \cdot \vec{r}\right) = \vec{F} \cdot \vec{v}$$

$$1\_Watt = 1\_J/s = 1\_kg \cdot m^2/s^3$$

$$1\_hp = 746\_W$$

$$1\_kWh = (1000\_W)(1\_h) = (1000\_W)(3600\_s) = 3.6 \times 10^5\_J$$