Physics I Lecture03-Motion in one dimension-I

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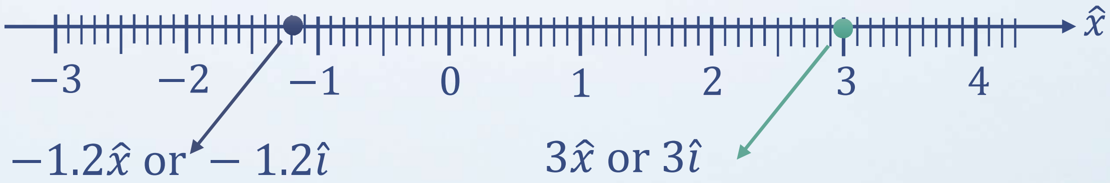
CONTENTS

1. POSITION, VELOCITY AND SPEED

Scalar: real number x, Vector: real number with direction $x\hat{\imath}$, where the number is just the length of the vector

Position – a vector to note the direction and distance from the origin



Displacement – a vector, variation of the position

Notation - $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$, where $\vec{x}_i$ and $\vec{x}_f$ are initial and final position

Example: The initial position of an object is $\vec{x}_i = 10\hat{\imath}$ and its final position is $\vec{x}_f = 4.2\hat{\imath}$. What is the displacement?

$$\Delta\vec{x} = \vec{x}_f - \vec{x}_i = 4.2\hat{\imath} - 10\hat{\imath} = -5.8\hat{\imath}$$

1. POSITION, VELOCITY AND SPEED

Distance – a scalar corresponding to the displacement

Notation - $|\Delta\vec{x}| = \left|\vec{x}_f - \vec{x}_i\right|$

Example: The initial position of an object is $\vec{x}_i = 10\hat{\imath}$ and its final position is $\vec{x}_f = 4.2\hat{\imath}$. What is the distance of its movement?

$$|\Delta\vec{x}| = \left|\vec{x}_f - \vec{x}_i\right| = |4.2\hat{\imath} - 10\hat{\imath}| = |-5.8\hat{\imath}| = 5.8$$

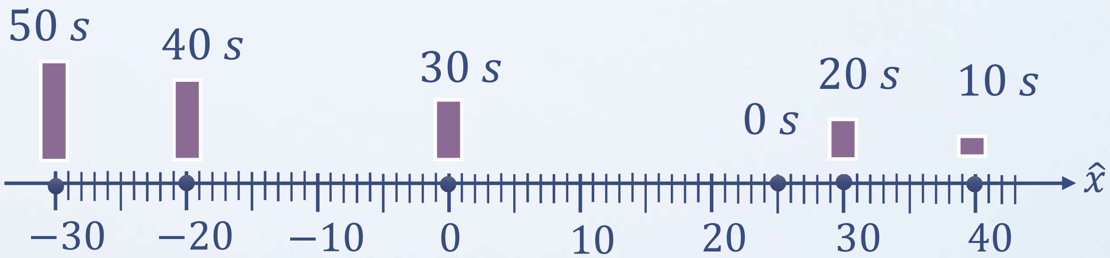
average Velocity – a vector, The displacement divides by the period of time.

Notation - $\vec{v}_{avg} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t}$

average Speed – a scalar, different from the average velocity.

Notation - $v_{avg} = \left|\frac{total\ distance\ traveled}{\Delta t}\right|$

1. POSITION, VELOCITY AND SPEED



t	x
0	25
10	40
20	30
30	0
40	-20
50	-30

average velocity between t=0 and t=50:

$$\vec{v}_{avg} = \frac{\vec{x}(50)-\vec{x}(0)}{50} = \frac{-30\hat{\imath}-25\hat{\imath}}{50} = -\frac{55}{50}\hat{\imath}\ (\mathrm{m/s})$$

average speed between t=0 and t=50:

$$v_{avg} = \frac{(40-2\ \ )\ \ (40-(-30))}{50} = \frac{85}{50}\ (\mathrm{m/s})$$

1. POSITION, VELOCITY AND SPEED

Example: A particle is moving along the x-axis. Its initial position is $\vec{x}_i = 12\hat{\imath}$ (m) at time $t_i = 1$ (s) and its final position is $\vec{x}_f = 2\hat{\imath}$ (m) at time $t_f = 4$ (s). Find out its displacement and average velocity during the time interval.

Displacement: $\Delta\vec{x} = \vec{x}_f - \vec{x}_i = (2 - 12)\hat{\imath} = -10\hat{\imath}$ (m)

Distance: $|\Delta\vec{x}| = |-10\hat{\imath}| = 10$ (n)

average Velocity: $\vec{v}_{avg} = {}^{\Delta\vec{x}}/_{\Delta t} = \frac{-10\hat{\imath}}{4-1} = -\frac{10}{3}\hat{\imath}$ (m/s)

2. INSTANTANEOUS VELOCITY AND SPEED

Velocity – a vector, The infinitesimal displacement divides by the infinitesimal period of time.

Notation -$\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{x}_f - \vec{x}_i}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t_f) - x(t_i)}{\Delta t} \hat{\imath} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \hat{\imath}$

$$= \frac{dx(t)}{dt} \hat{\imath}$$

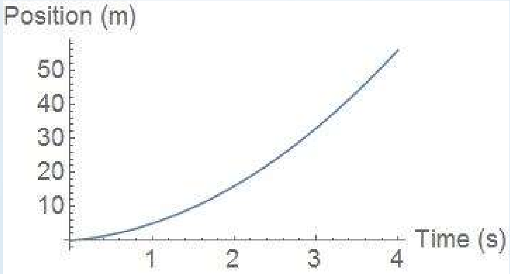
Speed – a scalar, The norm of the average velocity.

Notation - $v = |\vec{v}| = \left| \frac{dx(t)}{dt} \hat{\imath} \right| = \frac{dx(t)}{dt}$

http://588ku.com/image/qicheyibiaoban.html

2. INSTANTANEOUS VELOCITY AND SPEED

Example: The position of an object moving on the x-axis varies in time according to the equation $\vec{x}(t) = (3t^2 + 2t)\hat{\imath}$, where x is in meters and t is in seconds. (a) Find the velocity as a function of time. (b) Find the average velocity in the intervals between $t = 1$ and $t = 3$ s.



The velocity: $\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\vec{x}(t+\Delta t) - \vec{x}(t)}{\Delta t}$

$= \lim_{\Delta t \to 0} \frac{3(t+\Delta t)^2 + 2(t+\Delta t) - (3t^2+2t)}{\Delta t}\hat{\imath} = (6t + 2)\hat{\imath}$ (m/s)

The average velocity: $\vec{v}_{avg}(t) = \frac{\vec{x}(3) - \vec{x}(1)}{3-1} = \frac{33-5}{2}\hat{\imath} = 14\hat{\imath}$ (m/s)

Compared with $\vec{v}(1) = 8\hat{\imath}$ (m/s), $\vec{v}(2) = 14\hat{\imath}$ (m/s), $\vec{v}(3) = 20\hat{\imath}$ (m/s)

3. MOTION WITH CONSTANT VELOCITY

Object in constant velocity motion, its instantaneous velocity is $\vec{v}(t) = v_0\hat{\imath}$. As you know the velocity, you can find out its position as a function of time by integration with a specified constant of $\vec{x}(t=0) = \vec{x}(0) = \vec{x}_0$.

$$\vec{v}(t) = v_0\hat{\imath}$$

$$\frac{\vec{x}(t) - \vec{x}_0}{t - 0} = \vec{v}_{avg} = v_0\hat{\imath} \Rightarrow \vec{x}(t) - \vec{x}_0 = v_0 t\hat{\imath} \quad \vec{x}(t) = \vec{x}_0 + v_0 t\hat{\imath}$$

$$\frac{d\vec{x}(t)}{dt} = \vec{v}(t) = v_0\hat{\imath} \quad d\vec{x}(t) = v_0 dt\hat{\imath}$$

$$\int d\vec{x}(t) = \int v_0 dt\hat{\imath} \quad \int_{\vec{x}_0}^{\vec{x}(t)} d\big(\vec{x}(t')\big) = \int_0^t v_0 d(t')\hat{\imath}$$

$$\vec{x}(t) = \vec{x}_0 + v_0 t\hat{\imath}$$

3. MOTION WITH CONSTANT VELOCITY

Example: A particle moves with a constant velocity $\vec{v}(t) = 5.0\hat{\imath}$ (m/s).
The position is $\vec{x}(2.0) = 10\hat{\imath}$ (m) at $t = 2.0$ (s).
(a) Please find the position as a function of time.
(b) Please find its position at $t = 10$ (s).

$$\frac{d\vec{x}(t)}{dt} = \vec{v}(t) = 5.0\hat{\imath}$$

$$\int_{10\hat{\imath}}^{\vec{x}(t)} d\vec{x} = \hat{\imath}\int_{2.0}^{t} 5.0dt$$

$$\vec{x}(t) = \big(10 + 5.0(t - 2.0)\big)\hat{\imath} = 5.0t\hat{\imath}\ \text{(m)}$$

$$\vec{x}(10) = 50\hat{\imath}\ \text{(m)}$$

4. ACCELERATION

average Acceleration – a vector, The velocity variation divides by the period of time.

Notation - $\vec{a}_{avg} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

Acceleration – a vector, The infinitesimal velocity variation divides by the infinitesimal period of time.

Notation - $\vec{a} = \lim_{\Delta t \to 0} \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}(t_f) - \vec{v}(t_i)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t}$

Derivation - $\vec{a} = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{x}(t)}{dt^2}$

4. ACCELERATION

Example: A particle moves according to the expression $\vec{x}(t) = (4 - 27t + t^3)\hat{\imath}$, where x is in meters and t is in seconds. Please find its velocity and acceleration as a function of time.

$$\vec{v}(t) = (3t^2 - 27)\hat{\imath}\ \text{(m/s)}$$

$$\vec{a} = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{x}(t)}{dt^2}$$

$$\vec{a}(t) = (6t)\hat{\imath}\ \text{(m/s}^2\text{)}$$

https://www.pdhpe.net/the-body-in-motion/how-do-biomechanical-principles-influence-movement/motion/acceleration/

CONTENTS

5. MOTION DIAGRAM

car at rest

car in motion with constant velocity

car in motion with constant acceleration

car in motion with constant deceleration

https://giphy.com/gifs/dog-cartoons-UKm1AF0UrCkb6

6. MOTION WITH CONSTANT ACCELERATION

Object in constant acceleration motion, its instantaneous acceleration is $\vec{a}(t) = a_0\hat{\imath}$.
As you know the acceleration, you can find out its velocity and position as a function of time by integration with two specified constants of
$\vec{x}(t=0) = \vec{x}(0) = \vec{x}_0$ and $\vec{v}(t=0) = \vec{v}(0) = \vec{v}_0 = v_0\hat{\imath}$.

$$\vec{a}(t) = a_0\hat{\imath} = \vec{a}_{avg} = \frac{\vec{v}(t) - \vec{v}_0}{t - 0} \Rightarrow \vec{v}(t) - \vec{v}_0 = a_0 t\hat{\imath}$$

$$\vec{v}(t) = \vec{v}_0 + a_0 t\hat{\imath} = v_0\hat{\imath} + a_0 t\hat{\imath}$$

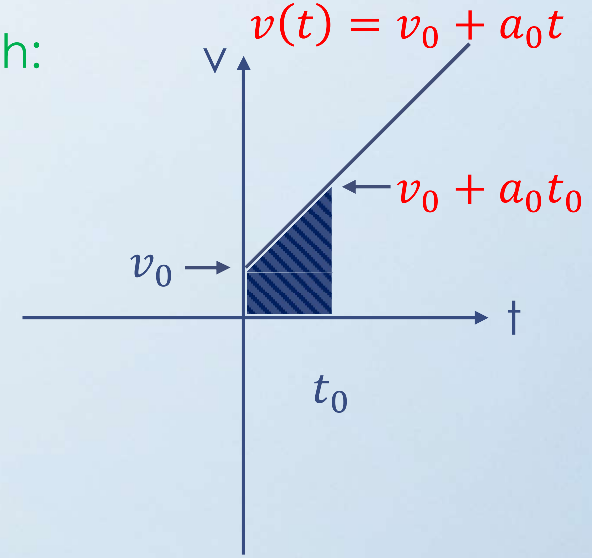
$$\frac{d\vec{v}(t)}{dt} = a_0\hat{\imath} \Rightarrow d\vec{v} = a_0\hat{\imath}dt \Rightarrow \int_{\vec{v}_0}^{\vec{v}(t)} d[\vec{v}] = \int_0^t a_0\hat{\imath}dt'$$

$$[\vec{v}]_{\vec{v}_0}^{\vec{v}(t)} = a_0\hat{\imath}[t']_0^t \Rightarrow \vec{v}(t) - \vec{v}_0 = a_0 t\hat{\imath} \Rightarrow \vec{v}(t) = \vec{v}_0 + a_0 t\hat{\imath}$$

6. MOTION WITH CONSTANT ACCELERATION

Object in constant acceleration motion, its instantaneous acceleration is $\vec{a}(t) = a_0\hat{\imath}$. As you know the acceleration, you can find out its velocity and position as a function of time by integration with two specified constants of $\vec{x}(t=0) = \vec{x}(0) = \vec{x}_0$ and $\vec{v}(t=0) = \vec{v}(0) = \vec{v}_0 = v_0\hat{\imath}$.

The area in v-t graph:



$$\vec{v}(t) = v_0\hat{\imath} + a_0 t\hat{\imath}$$

$$\vec{v}(0) = v_0\hat{\imath},\ \vec{v}(t_0) = v_0\hat{\imath} + a_0 t_0\hat{\imath}$$

$$x(t_0) - x(0) = \frac{v_0 + (v_0 + a_0 t_0)}{2} t_0 = v_0 t_0 + \frac{a_0 t_0^2}{2}$$

$$\Rightarrow x(t) = x_0 + v_0 t + \frac{a_0 t^2}{2}$$

$$\Rightarrow \vec{x}(t) = \vec{x}_0 + v_0 t\hat{\imath} + \frac{a_0 t^2}{2}\hat{\imath}$$

6. MOTION WITH CONSTANT ACCELERATION

Object in constant acceleration motion, its instantaneous acceleration is $\vec{a}(t) = a_0\hat{\imath}$. As you know the acceleration, you can find out its velocity and position as a function of time by integration with two specified constants of $\vec{x}(t=0) = \vec{x}(0) = \vec{x}_0$ and $\vec{v}(t=0) = \vec{v}(0) = \vec{v}_0 = v_0\hat{\imath}$.

$$\frac{d\vec{x}}{dt} = \vec{v}(t) = v_0\hat{\imath} + a_0 t\hat{\imath} \quad \Rightarrow \quad d\vec{x} = (v_0 + a_0 t\,)dt\hat{\imath}$$

$$\int_{\vec{x}_0}^{\vec{x}(t)} d(\vec{x}) = \int_0^t (v_0 + a_0 t)dt\,\hat{\imath}$$

$$\vec{x}(t) = \vec{x}_0 + v_0 t\hat{\imath} + \frac{a_0 t^2}{2}\hat{\imath}$$

6. MOTION WITH CONSTANT ACCELERATION

Three Equations: $\vec{v} = v_0\hat{\imath} + a_0 t\hat{\imath}$ —— 1st equation

$\vec{x} = \vec{x}_0 + v_0 t\hat{\imath} + \dfrac{a_0 t^2}{2}\hat{\imath}$ —— 2nd equation

$$\vec{v} - v_0\hat{\imath} = a_0 t\hat{\imath}$$

$$\vec{x} - \vec{x}_0 = v_0 t\hat{\imath} + \frac{a_0 t\hat{\imath}}{2} t \Rightarrow (\vec{x} - \vec{x}_0) a_0\hat{\imath} = v_0\hat{\imath} \cdot a_0 t\hat{\imath} + \frac{a_0 t\hat{\imath}}{2} \cdot a_0 t\hat{\imath}$$

$$\Delta\vec{x} \cdot \vec{a} = (v_0\hat{\imath}) \cdot (v\hat{\imath} - v_0\hat{\imath}) + \frac{1}{2}(v\hat{\imath} - v_0\hat{\imath}) \cdot (v\hat{\imath} - v_0\hat{\imath})$$

$v^2 = v_0^2 + 2\vec{a} \cdot \Delta\vec{x}$ —— 3rd equation

$$v^2 = v_0^2 + 2as$$

6. MOTION WITH CONSTANT ACCELERATION

Example: You start to brake your car from a speed of 108 to 72 km/h when spotting a police car. The traveled distance is 100 m. Assume that the car is in constant acceleration motion, please calculate its acceleration and the time required for the decrease in speed.

$$v_i = 108\frac{km}{h} \times \frac{1000m}{1km} \times \frac{1h}{3600s} = 30 \text{ (m/s)}$$

$$v_f = 72\frac{km}{h} \times \frac{1000m}{1km} \times \frac{1h}{3600s} = 20 \text{ (m/s)}$$

$$s = 100 \text{ (m)}$$

pick up the right equation: $v_f^2 = v_0^2 + 2as$

$$400 = 900 + 2a \times 100 \qquad a = -2.5 \text{ (m/s}^2\text{)}$$

pick up the right equation: $v_f = v_0 + at$

$$20 = 30 - 2.5 \times t \qquad t = 4 \text{ (s)}$$

6. MOTION WITH CONSTANT ACCELERATION

Example: An electron in the cathode-ray tube of a television set enters a region in which it accelerates uniformly in a straight line from a speed of 3×10^4 m/s to a speed of 5×10^6 m/s in a distance of 2 cm. How long is the electron in constant acceleration?

$v_i = 3 \times 10^4$ (m/s)

$v_f = 5 \times 10^6$ (m/s)

$s = 2$ (cm) $= 2 \frac{1m}{100cm} = 0.02$ (m)

pick up the right equation: $v_f^2 = v_0^2 + 2as$

$2.5 \times 10^{13} = 9 \times 10^8 + 2a \times 0.02$ $\quad a \cong 2.5 \times \frac{10^{13}}{0.04} = 6.25 \times 10^{14}$ (m/s²)

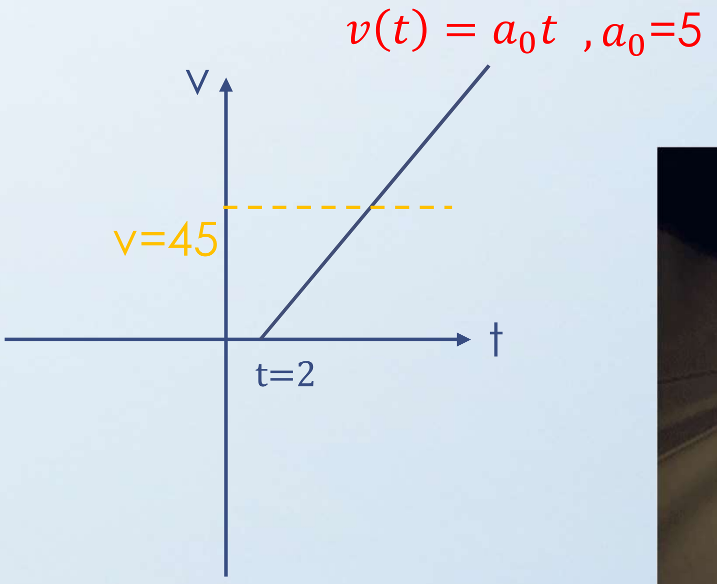
pick up the right equation: $v_f = v_0 + at$

$5 \times 10^6 = 3 \times 10^4 + 6.25 \times 10^{14} \times t$ $\quad t \cong 8 \times 10^{-9}$ (s)

6. MOTION WITH CONSTANT ACCELERATION

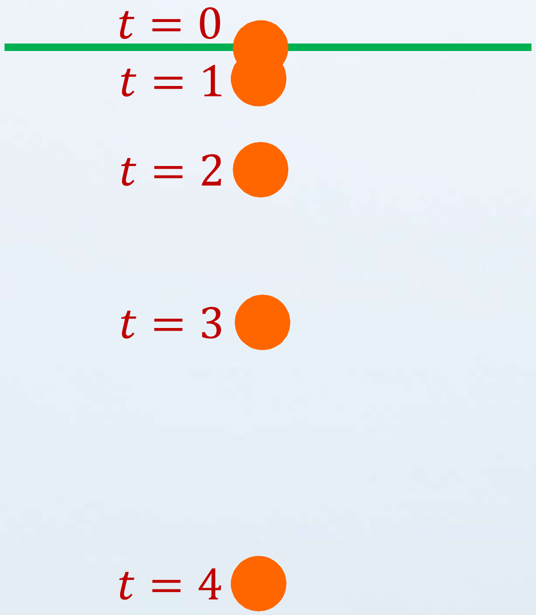
Example: A mortorcycle traveling at a constant speed of 45 m/s passes a trooper on a car hidden behind a billboard. 2 second after the speeding mortorcycle passes the billboard, the trooper sets out from the billboard to catch the mortorcycle, accelerating at a constant rate of 5.00 m/s^2. How long does it take her to overtake the mortorcycle?

$$45 \times 2 + 45t = \frac{5}{2}t^2$$



https://giphy.com/gifs/drive-fast-adrenaline-14hVD0HODUPY08

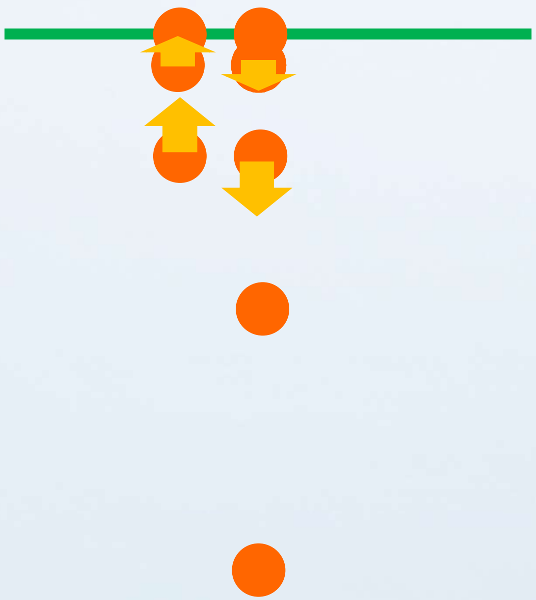
7. FREELY FALLING OBJECT



t	y(t) (m)	v_y(t) (m/s)	a(t) (m/s²)
0	0	0	-9.8
1	-4.9	-9.8	-9.8
2	-19.6	-19.6	-9.8
3	-44.1	-29.4	-9.8
4	-78.4	-39.2	-9.8
	-100		-9.8

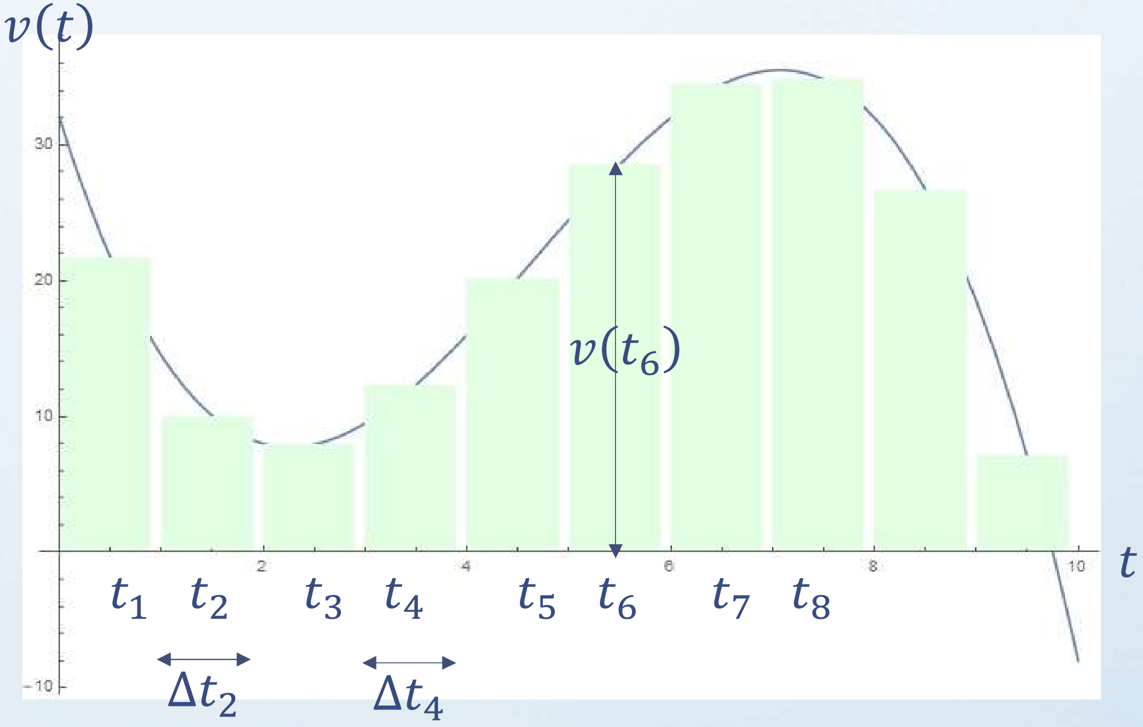
https://media.giphy.com/media/cXqQCO1bWp5tK/giphy.mp4

7. FREELY FALLING OBJECT



t	y(t) (m)	v_y(t) (m/s)	a(t) (m/s^2)
0	-19.6	19.6	-9.8
1	-4.9	9.8	-9.8
2	0	0	-9.8
3	-4.9	-9.8	-9.8
4	-19.6	-19.6	-9.8
5	-44.1	-29.4	-9.8
6	-78.4	-39.2	-9.8
	-100		-9.8

8. KINEMATIC EQUATIONS & CALCULUS



$$x = \lim_{\Delta t_n \to 0} \sum_{n=1}^{m} v(t_n)\Delta t_n$$

$$x(t_m) - x(0) = \int_0^{t_m} v(t)dt$$

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