Lecture 03 Motion in One Dimension

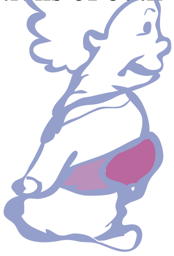
This lecture gives you the same content as that in **Chapter 02 in Serway/Jewett's** textbook of "Physics for Scientists and Engineers with Modern Physics".

Particle model: a particle is a point-like object, that is, an object that has mass but is of infinitesimal size

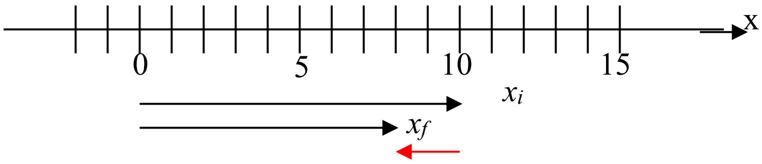
3.1 Positon, Velocity, and Speed

A vector quantity requires the specifications of both direction and magnitude

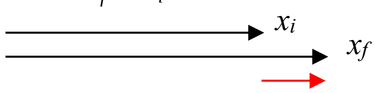
vector: displacement: $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$



nonsymmetry nature

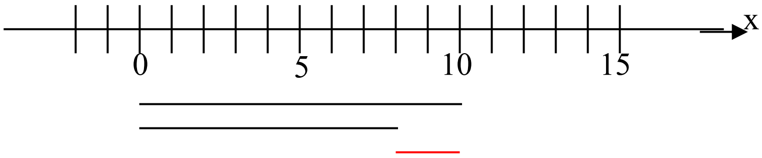


$\vec{x}_i = 10\hat{i}$, $\vec{x}_f = 8\hat{i}$ --> $\Delta\vec{x} = \vec{x}_f - \vec{x}_i = 8\hat{i} - 10\hat{i} = -2\hat{i}$

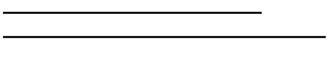


$\vec{x}_i = 8\hat{i}$, $\vec{x}_f = 10\hat{i}$ --> $\Delta\vec{x} = \vec{x}_f - \vec{x}_i = 10\hat{i} - 8\hat{i} = 2\hat{i}$

scalar: distance: $|\Delta\vec{x}| = |\vec{x}_f - \vec{x}_i|$ (no negative sign)



$\vec{x}_i = 10\hat{i}$, $\vec{x}_f = 8\hat{i}$ --> $\Delta\vec{x} = \vec{x}_f - \vec{x}_i = 8\hat{i} - 10\hat{i} = -2\hat{i}$ --> $|\Delta\vec{x}| = 2$



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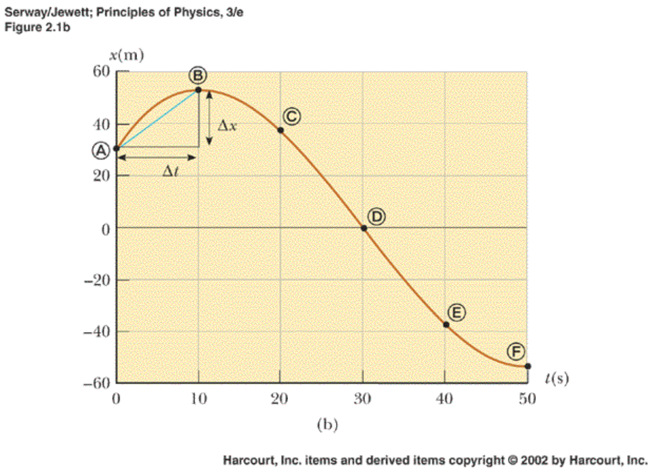
vector: average velocity: $\vec{v}_{avg} = \dfrac{\Delta\vec{x}}{\Delta t}$

scalar: average seed: $v_{avg} = \dfrac{|\Delta\vec{x}|}{\Delta t}$, $Average\_Speed = \dfrac{total\_distance}{total\_time}$



Example: A particle moving along the x axis is located at xi = 12 m at ti = 1s and xf = 4 m at tf = 3 s. Find its displacement and average velocity during this time interval.

Displacement: $\Delta\vec{x} = 4 - 12 = -8\,\text{m}$, distance: $\Delta x = |\Delta\vec{x}| = |4-12| = 8\,\text{m}$

Average velocity: $\vec{v}_{avg} = \dfrac{x_f - x_i}{t_f - t_i} = \dfrac{4-12}{3-1} = -4\,\text{m/s}$, Average speed: 4 m/s

What is position?

3.2 Instantaneous Velocity and Speed

vector: velocity: $\vec{v} = \lim_{\Delta t \to 0} \dfrac{\Delta\vec{x}}{\Delta t} = \dfrac{d}{dt}\vec{x}$

scalar: speed: $|\vec{v}| = \lim_{\Delta t \to 0} \dfrac{|\Delta\vec{x}|}{\Delta t} = \left|\dfrac{d}{dt}\vec{x}\right|$

Example: The position of a particle moving along x axis varies in time according to the expression $\vec{x} = 3t^2\hat{i}$, where x is in meters and t is in seconds. Find the velocity in terms of t at any time. Find the average velocity in the intervals t = 0 s to t = 2 s.

$\vec{x}_i = \vec{x}(t) = 3t^2\hat{i}$, $\vec{x}_f = \vec{x}(t+\Delta t) = 3(t+\Delta t)^2\hat{i}$

$$\vec{v} = \lim_{\Delta t->0}\frac{\vec{x}_f - \vec{x}_i}{\Delta t} = \lim_{\Delta t->0}\frac{3t^2 + 6t\Delta t + 3(\Delta t)^2 - 3t^2}{\Delta t}\hat{i} = 6t\hat{i}$$

$$\vec{v}_{avg} = \frac{\vec{x}(t_2) - \vec{x}(t_1)}{t_2 - t_1} = \frac{3*2^2 - 3*0^2}{2-0}\hat{i} = 6 \ \text{ m/s}$$

3.3 Analysis Model: The Particle Under Constant Velocity

$$\left(\frac{d}{dt}\right)\vec{x} = \vec{v}_0 = v_0\hat{\imath}$$

$$(d\vec{x}) = v_0 dt\hat{\imath}$$

$$\int d(\vec{x}) = \hat{\imath}v_0\int dt$$

$$\int_{\vec{x}_0}^{\vec{x}(t')} d(\vec{x}) = \hat{\imath}v_0\int_0^{t'} dt$$

$$[\vec{x}]_{\vec{x}_0}^{\vec{x}(t')} = \hat{\imath}v_0[t]_0^{t'}$$

$$\vec{x}(t') - \vec{x}_0 = \hat{\imath}v_0(t'-0)$$

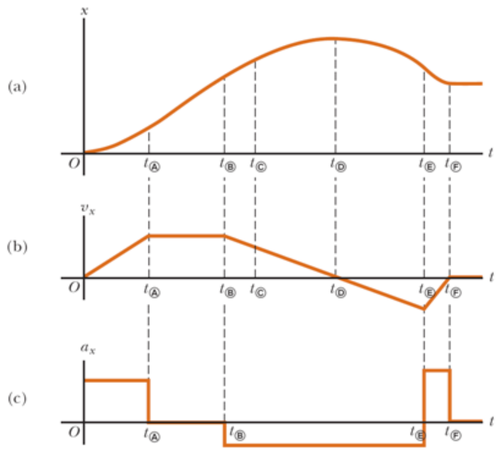
$$\vec{x}(t') = \vec{x}_0 + \hat{\imath}v_0 t'$$

$$\vec{x}(t) = \vec{x}_0 + \vec{v}_0 t$$

Example: A particle move at a constant velocity $\vec{v} = 5\cdot\hat{i}(m/s)$, the initial position xi = 10 m, find the final position after a time interval of t = 10 s.

$\vec{x}_f = \vec{x}_i + \vec{v}t$ --> $\vec{x}_f = (10 + 5*10)\hat{i} = 60\hat{i}(m)$

3.4 Acceleration



a: Position, b: velocity, c: acceleration

vector: average acceleration: $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$

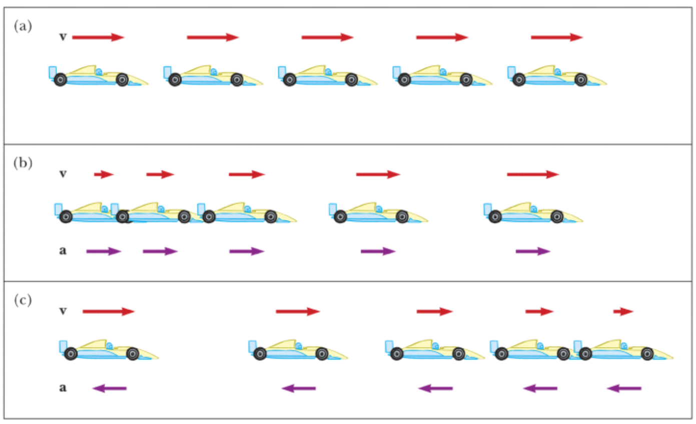
vector: instantaneous acceleration: $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta r} = \frac{d}{dt}\vec{v} = \left(\frac{d}{dt}\right)\left(\frac{d}{dt}\vec{x}\right) = \frac{d^2}{dt^2}\vec{x}$

Example: A particle's position on the x axis is given by $x = 4 - 27t + t^3$ with x in meters and t in seconds. (a) Find the particle's velocity function v(t) and acceleration function a(t).

$v(t) = \frac{dx(t)}{dt} = 0 - 27 + 3t^2$, $a(t) = \frac{dv(t)}{dt} = 6t$

3.5 Motion Diagrams

Serway/Jewett; Principles of Physics, 3/e
Figure 2.11



3.6 Motion Under Constant Acceleration

$$\vec{a}(t) = \vec{a}_0 = \vec{a}_{avg}$$

$$\vec{a}_0 = \vec{a}_{avg} = \frac{\vec{v}(t) - \vec{v}_0}{t - 0}$$

$$\vec{v}(t) = \vec{v}_0 + \vec{a}_0 t$$ ***- The 1st formula***

$$\frac{d\vec{x}}{dt} = \vec{v}(t) = \vec{v}_0 + \vec{a}_0 t$$

$$d\vec{x} = (\vec{v}_0 + \vec{a}_0 t)dt$$

$$\int_{\vec{x}_0}^{\vec{x}(t')} d(\vec{x}) = \int_0^{t'} (\vec{v}_0 + \vec{a}_0 t)dt$$

$$\vec{x}(t') - \vec{x}_0 = \vec{v}_0 t' + \vec{a}_0 (t'^2/2)$$ ***- The 2nd formula***

Use the first formula, $\vec{a}_0 t = \vec{v}(t) - \vec{v}_0$.

Multiply the second formula by dot product with $\vec{a}_0$

$$\vec{a}_0 \cdot (\vec{x}(t) - \vec{x}_0) = \vec{v}_0 \cdot \vec{a}_0 t + \frac{1}{2}(\vec{a}_0 t) \cdot (\vec{a}_0 t)$$

$$\vec{a}_0 \cdot (\vec{x}(t) - \vec{x}_0) = \vec{v}_0 \cdot (\vec{v}(t) - \vec{v}_0) + \frac{1}{2}(\vec{v}(t) - \vec{v}_0) \cdot (\vec{v}(t) - \vec{v}_0)$$

$$\vec{a}_0 \cdot (\vec{x}(t) - \vec{x}_0) = \vec{v}_0 \cdot \vec{v}(t) - v_0^2 + \frac{1}{2}v^2 - \vec{v}_0 \cdot \vec{v}(t) + \frac{1}{2}v_0^2$$

$$\vec{a}_0 \cdot (\vec{x}(t) - \vec{x}_0) = \frac{1}{2}v^2 - \frac{1}{2}v_0^2$$

$$v^2 = v_0^2 + 2\vec{a}_0 \cdot (\vec{x}(t) - \vec{x}_0)$$ ***- The 3rd formula***

Example: Spotting a police car, you brake a Porsche from a speed of 100 km/h to a speed of 80.0 km/h during a displacement of 88.0 m, at a constant acceleration. (a) What is that acceleration? (b) How much time is required for the given decrease in speed?

$\vec{v}_f = \vec{v}_0 + \vec{a}t$, $\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$, $v^2 = v_0^2 + 2\vec{a} \cdot \Delta\vec{x}$

$v_f = 80\frac{km}{h}1000\frac{m}{km}\frac{1}{3600}\frac{h}{s} = 22.2$ m/s, $v_i = 100\frac{1000}{3600} = 27.8$ m/s

$\Delta\vec{x} = 88$ m -> choose the 3rd formula, $22.2^2 = 27.8^2 + 2 * a * 88$, a=-1.6 m/s2

Example: Accelerating an Electron

An electron in the cathode-ray tube of a television set enters a region in which it accelerates uniformly in a straight line from a speed of 3 x 104 m/s to a speed of 5 x 106 m/s in a distance of 2 cm. For what length of time is the electron accelerating?

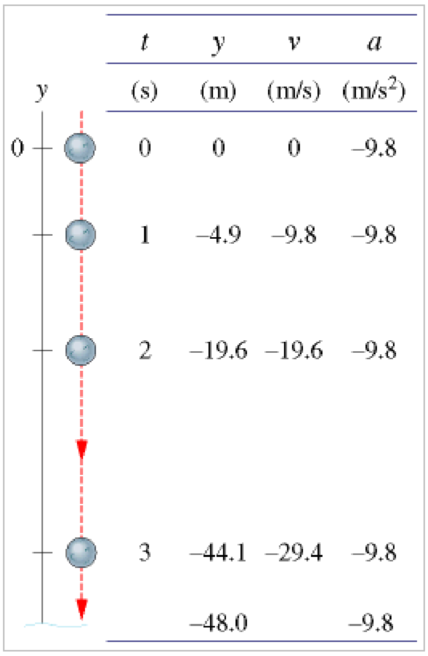
Choose the 3rd formula: $\left(5\times10^{6}\right)^{2}=\left(3\times10^{4}\right)^{2}+2a*\frac{2}{100}$, a=6.2*1014 m/s2

Choose the 1st formula: $\Delta t=\frac{5\times10^{6}-3\times10^{4}}{6.2\times10^{14}}=8.0\times10^{-9}\,\mathrm{s}$

Example: A car traveling at a constant speed of 45.0 m/s passes a trooper on a mortorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.00 m/s2. How long does it take her to overtake the car?

$$45+45t=\frac{1}{2}3t^{2}$$

3.7 Freely Falling Objects



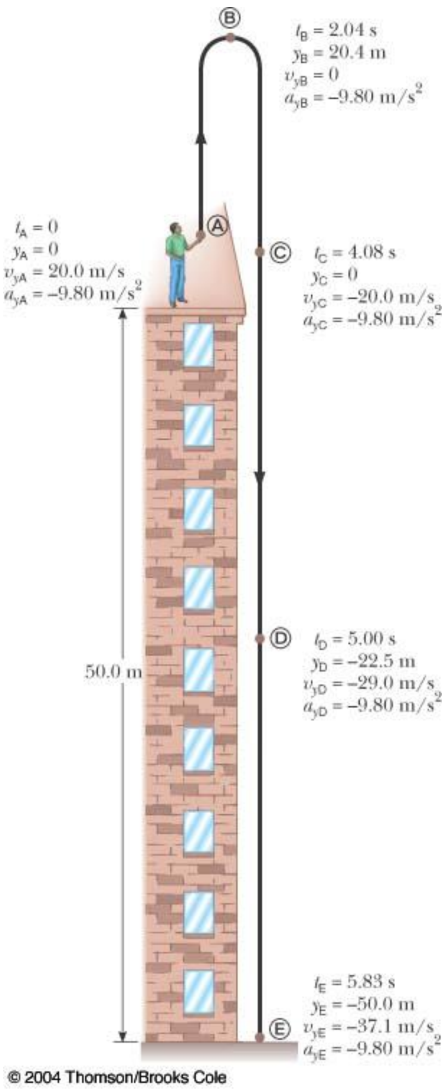
up: +y

$$\vec{a}=-g=-9.8\frac{m}{s^{2}}$$

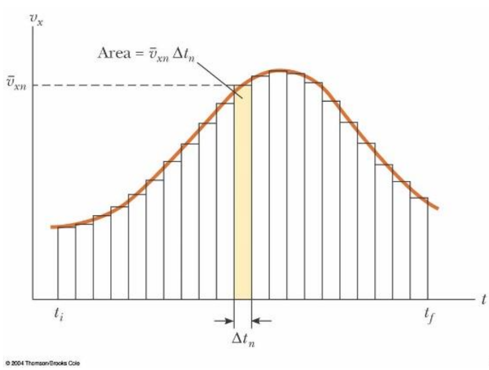
t

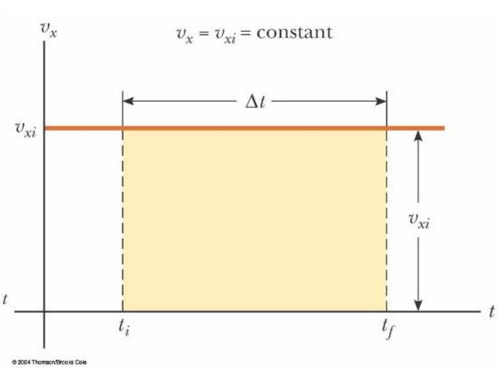
$$a=a_0$$
$$v=v_0+at=a_0t$$
$$x=x_0+v_0t+\frac{1}{2}at^{2}=\frac{1}{2}a_0t^{2}$$

Example: A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Fig. 2.14. Using tA = 0 as the time the stone leaves the thrower's hand at position A, determine (A) the time at which the stone reaches its maximum height, (B) the maximum height, (C) the time at which the stone returns to the height from which it was thrown, (D) the velocity of the stone at this instant, and (E) the velocity and position of the stone at t = 5.00 s.



3.8 Kinematic Equations Derived from Calculus





$$\Delta x = \sum_n \bar{v}_{xn} \Delta t_n = \lim_{\Delta t -> 0} \sum_n \bar{v}_{xn} \Delta t_n = \int_{ti}^{tf} v_x(t) dt$$ (What's definite & indefinite integral ?)

$$v_x = \frac{dx}{dt} \text{ -> } dx = v_x dt \text{ -> } \int_{x1}^{x2} dx = \int_{t1}^{t2} v_x dt$$

$$a_x = \frac{dv_x}{dt} \text{ -> } v_{xf} - v_{xi} = \int_0^t a_x dt$$

For the case of constant acceleration, we can derive the kinematic equations:

$$\frac{dv}{dt} = a_0 = const. \text{ -> } v = v_0 + at$$

$$v = \frac{dx}{dt} = v_0 + at \text{ -> } x = x_0 + v_0 t + \frac{1}{2} a t^2$$