## Lecture 05 The Laws of Motion

We know how to express the position, velocity, and acceleration.
What is the mechanism that changes the state of motion?

First Law: An object at rest stays at rest unless acted on by an external force. An object in motion continues to travel with constant velocity unless acted on by an external force.

Second Law: The direction of the acceleration of an object is in the direction of the net external force acting on it. The acceleration is proportional to the net external force $\vec{F}_{\text {net }}$, in accordance with $\vec{F}_{n e t}=m \vec{a}$, where $m$ is the mass of the object. The net force acting on an object, also called resultant force, is the vector sum of all the forces acting on it: $\vec{F}_{n e t}=\sum \vec{F}$. Thus, $\sum \vec{F}=m \vec{a}$.

Third Law: Forces always occur in equal and opposite pairs. If object A exerts a force $\vec{F}_{A, B}$ on object B, an equal but opposite force $\vec{F}_{B, A}$ is expected by object B on object
A. Thus, $\vec{F}_{B, A}=-\vec{F}_{A, B}$.

### 5.1 The Concept of Force

What do you experience the concept of force? Muscular activity?

We need to know the relation between the force on the object and the change in motion of that object.
What are contact force and field force?
What is the net force? Use the rules of vector addition to obtain the net force on an object.


What kind of force will cause the motion of constant velocity?
What kind of force will cause the motion of constant speed?
What kind of force will cause the motion of constant acceleration?

If the net force exerted on an object is zero, the acceleration of the object is zero and its velocity remains constant.

Does zero velocity means being in equilibrium??

How do we measure the strength of force?

## The Vector Nature of Force

### 5.2 Newton's First Law and Inertia Frames

Inertia, Law of Inertia, First Law

in the absence of external force, an object at rest remains at rest and an object in motion continues in motion with a constant velocity

1. Inertia frame of reference is one in which Newton's first law is valid.
2. Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame.
3. If no forces act on an object, any reference frame with respect to which the acceleration of the object remains zero is an inertial reference frame.

## Examples of non-inertial:

1. the accelerating train is not an inertia frame - ghost force
2. the earth is not an inertia frame

### 5.3 Mass

Inertia Mass: is the measure of an object's resistance to a change in motion in response to an external force
a force acting on $\mathrm{m}_{1}\left(\mathrm{~m}_{2}\right)$ produces an acceleration $\mathrm{a}_{1}\left(\mathrm{a}_{2}\right), \frac{m_{1}}{m_{2}}=\frac{a_{2}}{a_{1}}$
Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it.
Mass and weight are two different quantities

### 5.4 Newton's Second Law

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.
$\vec{a}=\frac{\sum \vec{F}}{m}$
unit of force: Newton, $1 \mathrm{~N}=1 \mathrm{~kg} \mathrm{~m} / \mathrm{s}^{2}$

| System of <br> units | Mass | Acceleration | Force |
| :--- | :--- | :--- | :--- |
| SI | kg | $\mathrm{m} / \mathrm{s}^{2}$ | $\mathrm{~kg} \mathrm{~m} / \mathrm{s}^{2}$ |
| US customary | slug | $\mathrm{ft} / \mathrm{s}^{2}$ | slug ft/s ${ }^{2}$ |

Example: A 0.3 kg hockey puck slides on the horizontal frictionless surface of an ice rink. It is struck simultaneously by two different hockey sticks. The forces are shown in figure. Determine the acceleration of the puck.

$\vec{F}=\left(5.0 \cos \left(20^{\circ}\right) \hat{i}-5.0 \sin \left(20^{\circ}\right) \hat{j}\right)+\left(8.0 \cos \left(60^{\circ}\right) \hat{i}+8.0 \sin \left(60^{\circ}\right)^{\prime \prime} \hat{j}\right)$
$a=|\vec{F}| / m$

### 5.5 The Gravitation Force and Weight

The magnitude of gravitational force is called the weight of an object: $\vec{F}_{g}=m \vec{g}$.
Objects weigh less at higher altitudes than at sea level.
weight $=$ ? mass
inertia mass $\longleftrightarrow \rightarrow$ gravitational mass
Example: If you want to construct a ladder from the earth to the moon and the gravitation varies as a function of $\mathrm{r}-2$, how much weight shall you support on the ground?

$$
\frac{\Delta M}{\Delta h}=\rho, g(h)=\frac{R^{2}}{(R+h)^{2}} g,
$$


$\int_{0}^{R} g(h) \cdot d M=\int_{0}^{R} \frac{R^{2}}{(R+h)^{2}} g \cdot \rho \cdot d h=\int_{0}^{R} \rho g R^{2} d\left[-(h+R)^{-1}\right]=\rho g R^{2} \frac{1}{2 R}=\frac{1}{2} \rho g R$
$R \sim 6.4 \cdot 10^{6} \mathrm{~m}, \quad \rho \sim 1 \cdot \mathrm{~kg} / \mathrm{m}, \quad g \sim 10 \cdot \mathrm{~m} / \mathrm{s}^{2}$
pressure: $6.4 \cdot 10^{7} \cdot \mathrm{nt} / 1 \cdot \mathrm{~m}^{2}=6.4 \cdot 10^{3} \mathrm{Nt} / \mathrm{cm}^{2}, 1 \cdot \mathrm{~atm} \sim 10 \cdot \mathrm{nt} / \mathrm{cm}^{2}$
Only carbon nanotube can sustain with this high pressure.

### 5.6 Newton's Third Law

$\vec{F}_{12}=-\vec{F}_{21}$

1. Forces always occur in pairs.
2. The action force is equal in magnitude to the reaction force and opposite in direction.
3. The two forces in an action-reaction pair always act
 on two different objects..


Example: If a sports car collides head-on with a massive truck, which vehicle experiences the greater force? Which vehicle experiences the greater acceleration?

## Forces in Natures - The Fundamental Forces:

1. The Gravitational Forces - the force between objects
2. The Electromagnetic Forces - the force between electric charges
3. The Strong Nuclear Forces - the force between subatomic particles
4. The Weak Nuclear Forces - during certain radioactive decay process


Ref: http://www.fotosearch.com/DGT307/pil0054/
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## Action at a Distance $\leftarrow \rightarrow$ Contact Forces

Displaying the force for solids and springs?

(a)

### 5.7 Some Applications of Newton's Laws

When we applied Newton's law to an object, we are interested only in external forces acting on the object.

How do we solve this problem -> Separate it into components in orthogonal directions


## Objects in Equilibrium

If the acceleration of an object is zero, the particle is in equilibrium.
$\sum \vec{F}=0 \rightarrow \sum F_{i x}=0, \quad \sum F_{i y}=0$, and $\sum F_{i z}=0$
Is the egg sitting on the ground in equilibrium?
Is the flying man in equilibrium in the space ?
What is the free-body diagram?

Objects Experiencing a Net Force
And finally transform to acceleration

What's the tension?


What ste tension?


Example: a traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in figure.
$\vec{T}_{1}=-T_{1} \cos 37^{\circ} \hat{i}+T_{1} \sin 37^{\circ} \hat{j}$
$\vec{T}_{2}=T_{2} \cos 53^{\circ} \hat{i}+T_{2} \sin 53^{\circ} \hat{j}$
$T_{1}=73.2 \cdot N, T_{2}=97.6 \cdot N$


Example: As your jet plane speeds down the runway on takeoff, you decide to determine its acceleration, so you take out your yo-yo and note that when you suspend it, the string makes an angle of $22^{\circ}$ with the vertical. (a) What is the acceleration of the plane? (b) If the mass of the yo-yo is 40 g , what is the tension in the string?

The tension in the string provides acceleration for the yo-yo.
(a) $\frac{a}{g}=\tan \left(22^{\circ}\right)$
(b) $T \cos \left(22^{\circ}\right)=m g$


Example: A person weighs a fish of mass $m$ on a spring scale attached to the ceiling of an elevator. Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.


Example: The Atwood Machine

$$
\begin{aligned}
& -m_{2} g \cdot \hat{j}+\vec{T}=m_{2} a \cdot \hat{j} \\
& -m_{1} g \cdot \hat{j}+\vec{T}=-m_{1} a \cdot \hat{j} \\
& \left(m_{1}+m_{2}\right) a=\left(m_{1}-m_{2}\right) g \\
& a=\frac{m 1-m 2}{m 1+m 2} g
\end{aligned}
$$


(b)

Example: A constant F is applied on two blocks with mass $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$, respectively. Determine the action and reaction forces between them.

$$
\vec{F}=F \cdot \hat{i}, \quad \vec{a}=\frac{F}{M_{1}+M_{2}} \hat{i}, \quad \vec{F}_{12}=M_{2} \cdot \vec{a}=\frac{M_{2}}{M_{1}+M_{2}} F \cdot \hat{i}
$$


(a)

Example: A ball of mass $m_{1}$ and a block of mass $m_{2}$ are attached by a lightweight cord that passes through a frictionless pulley of negligible mass. The block lies on a frictionless incline of angle $\theta$. Find the magnitude of the acceleration of the two objects and the tension in the cord.

(a)

What's the force direction given by the incline? (the incline force you to use the new coordinate with x and y direction parallel and perpendicular to the incline)
What's does the incline do for the motion?

$$
\begin{aligned}
& m_{2} g \sin \theta-T=m_{2} a \\
& T-m_{1} g=m_{1} a \\
& -->m_{2} g \sin \theta-m_{1} g=\left(m_{1}+m_{2}\right) a \quad->\quad a=\frac{m_{2} \sin \theta-m_{1}}{m_{1}+m_{2}} g
\end{aligned}
$$

Example: The figure shows a block $S$ (the sliding block) with mass $\mathrm{M}=3.3 \mathrm{~kg}$. The block is free to move along a horizontal frictionless surface such as an air table. This first block is connected by a cord that wraps over a frictionless pulley to a second block $H$ (the hanging block), with mass $\mathrm{m}=2.1 \mathrm{~kg}$. The cord and pulley have negligible masses compared to the blocks (they are "massless"). The hanging block $H$ falls as the sliding block $S$ accelerates to the right. Find (a) the acceleration of the sliding block, (b) the acceleration of the hanging block, and (c) the tension in the cord.
$-m g \cdot \hat{k}+T \cdot \hat{k}=-m a \cdot \hat{k}$
$T \cdot \hat{i}=M a \cdot \hat{i}$
$a=\frac{m g}{M+m}=3.81 \cdot \mathrm{~m} / \mathrm{s}^{2}$
$T=13 \cdot N$


### 5.8 Forces of Friction

Normal Force: $\vec{N}, \vec{F}_{N}$


Static Friction: $\quad \vec{F}_{s, \text { max }}=\mu_{s} \vec{N}$
coefficient of static friction: $f_{s} \leq N \mu_{s}$

Kinetic Friction: $\quad \vec{F}_{k, \text { max }}=\mu_{k} \vec{N}$
coefficient of kinetic friction: $f_{k}=N \mu_{k}$

Rolling Friction: $\quad \vec{F}_{r, \text { max }}=\mu_{r} \vec{N}$

What can the two types of friction do? - inertial movement
When you turn on an air conditioner, do you need to supply a huge current?
When you turn on a motor...

## Friction Explained:

Friction is a complex, incompletely understood phenomena that arises from the attraction of molecules between two surfaces that are in close contact.



Approximation Values of Friction Coefficients

| Materials | $\mu_{s}$ | $\mu_{k}$ | Materials | $\mu_{s}$ | $\mu_{k}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Steel on steel | 0.7 | 0.6 | Glass on glass | 0.9 | 0.4 |
| Brass on steel | 0.5 | 0.4 | Teflon on teflon | 0.04 | 0.04 |
| Copper on cast iron | 1.1 | 0.3 | Teflon on steel | 0.04 | 0.04 |

Example: Suppose a block is placed on a rough surface $\qquad$ inclined relative to the horizontal. (a) How is the coefficient of static friction related to the critical angle $\theta_{c}$ at which the block begins to move? (b) How could we find the coefficient of kinetic fricition?
(a) $m g \sin \theta_{c}=\mu_{s} m g \cos \theta_{c}$
(b) $F=m g \sin \theta-\mu_{k} m g \cos \theta=m a, \quad h=\frac{1}{2} a t^{2}$

determine the time required to move down from a hight of $h$

Example: A block of mass $\mathrm{m}_{1}$ on a rough, horizontal surface is connected to a ball of mass $\mathrm{m}_{2}$ by a lightweight cord over a lightweight, frictionless pulley. A force of magnitude F at an angle of $\theta$ with the horizontal is applied to the block as shown. The coefficient of kinetic friction between the block and surface is $\mu_{\mathrm{k}}$. Determine the magnitude of the acceleration of the two objects.
 What's the normal force, $\vec{N}$ ? Is the block moving or at rest? Do they move together?

$$
f_{s}=\left(m_{1} g-F \sin \theta\right) \mu_{k},\left(m_{1}+m_{2}\right) a=F \cos \theta-f_{s}
$$

Example: A child of mass $m_{c}$ sits on a toboggan of mass $m_{t}$, which in turn sits on a frozen pond assumed to be frictionless. The toboggan is pulled with a horizontal force $\vec{F}$. The coefficients of static and kinetic sliding friction between the child and toboggan are $\mu_{s}$ and $\mu_{k}$. (a) Find the maximum value of F for which the child will not slide relative to the toboggan. (b) Find the acceleration of the toboggan and child when F is greater than this value.
(a) $F_{c}=m_{c} g \mu_{s}=m_{c} a_{c}, \quad F_{\text {total }}=\left(m_{c}+m_{t}\right) a_{c}=\left(m_{c}+m_{t}\right) \mu_{s} g$

(b) $F-f_{k}=m_{t} a_{t}, f_{k}=m_{c} g \mu_{k}=m_{c} a_{c}$
$a_{c}=g \mu_{k}, \quad a_{t}=\frac{F-\mu_{k} m_{c} g}{m_{t}}$


