Lecture 12 Static Equilibrium Elasticity

12.1 The Rigid Object in Equilibrium

1. The net external force acting on the body must remain zero:

$$\sum \vec{F}_i = 0$$

2. The net external torque about any point must remain zero:

$$\sum \vec{\tau}_i = 0$$

$\vec{F} = 0$, $\dfrac{d\vec{P}}{dt} = \vec{F} = 0$, $\vec{P} = const$, and

$\vec{\tau} = 0$, $\dfrac{d\vec{L}}{dt} = \vec{\tau} = 0$, $\vec{L} = const$ → such objects are in equilibrium

We shall simplify matters by considering only situations in which the forces that act on the body lie in the xy plane. This means that the only torques that can act on the body must tend to cause rotation around an axis parallel to the z axis. With this assumption, we eliminate one force equation and two torque equations.

12.2 More on the Center of Gravity

let $\vec{W}$ be the center of gravity

$$\vec{\tau}_{net} = \vec{r}_{COG} \times \vec{W}$$

if $\vec{g}$ is the same for all elements of a body, then the body's center of gravity is coincident with the body's center of mass

Proof:

$\tau_i = x_i F_{gi}$, $\tau = \sum \tau_i = \sum x_i F_{gi} = x_{cog} F_g$, $F_{gi} = m_i g_i$, $F_g = \sum m_i g_i$

if $g_i = g$, $(\sum x_i m_i g) = x_{cog}(\sum m_i g)$, $x_{cog} = \dfrac{\sum m_i x_i}{\sum m_i}$

12.3 Examples of Rigid Objects in

Equilibrium

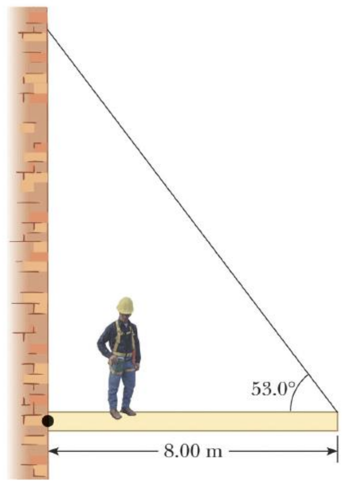
$\sum \vec{F} = 0$, $\sum \vec{\tau} = 0$

Example:

Standing on a horizontal beam

A uniform horizontal beam of length 8 m and weight 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 53° with the horizontal. If a 600-N man stands 2 m from the wall, find the tension in the cable and the force exerted by the wall on the beam.

Serway/Jewett; Principles of Physics, 3/e
Figure 10.17a



(a)

$$8 \cdot T \cdot \sin 53^o = 4 \cdot 200 + 2 \cdot 600$$

$$T = 313N$$

$$F_x - T\cos 53^o = 0$$

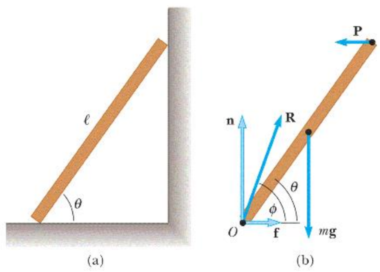
$$F_y + T\sin 53^o - 200 - 600 = 0$$

$$F_x = 188N\,,\quad F_y = 550$$

Example: The leaning ladder

A uniform ladder of length l and mass m rests against a smooth, vertical wall. If the coefficient of static friction between ladder and the ground is $\mu_s = 0.4$, find the minimum angle $\theta_{\min}$ such that the ladder does not slip.

Serway/Jewett; Principles of Physics, 3/e
Figure 10.18



$$P = f\,,\quad N = mg$$

$$mg\frac{l}{2}\cos\theta = Pl\sin\theta$$

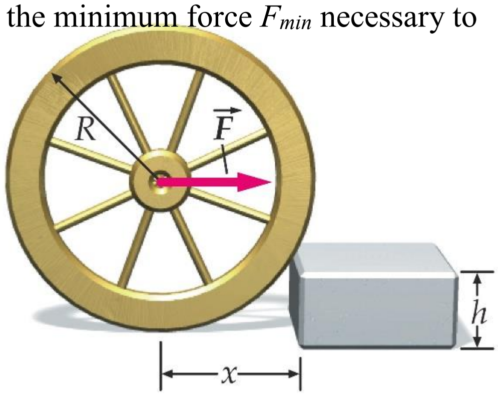
$$f = N\mu_s = 0.4n$$

$$\tan\theta_{\min} = \frac{mgl}{2Pl} = \frac{mg}{2P} = \frac{mg}{2f} = \frac{mg}{2N\mu_s} = \frac{1}{2\mu_s}$$

$$\tan\theta_{\min} = 1.25\,,\quad \theta_{\min} = 51^o$$

$$\theta \ge \theta_{\min} \rightarrow mgR\cos\theta/2 \le PR\sin\theta\,;\quad \theta < \theta_{\min} \rightarrow mgR\cos\theta/2 > PR\sin\theta$$

Example: A wheel of mass M and radius R rests on a horizontal surface against a step of height h (h < R). The wheel is to be raised over the step by a horizontal force $\vec{F}$ applied to the axle of the wheel as shown. Find the minimum force F_{min} necessary to raise the wheel over the step.



$$F_{\min}(R-h)=Mgx$$

$$x=\sqrt{R^2-(R-h)^2}$$

$$F_{\min}=$$

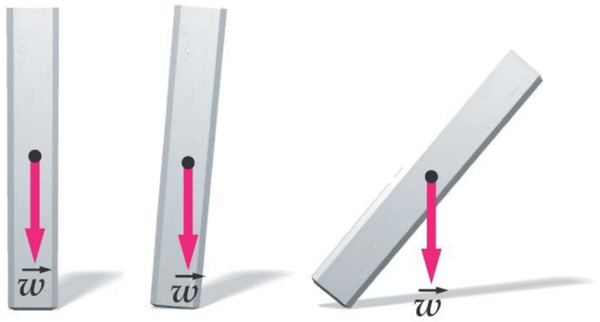
Couples

Two forces that are equal and opposite are called a couple.
The torque produced by this couple about an arbitrary point O is

$\vec{\tau}=\vec{r}_1\times\vec{F}_1+\vec{r}_2\times\vec{F}_2=\vec{r}_1\times\vec{F}_1-\vec{r}_2\times\vec{F}_1=(\vec{r}_1-\vec{r}_2)\times\vec{F}_1$, do not depend on the choice of O.

The torque produced by a couple is the same about all points in space.

Stability of Rotational Equilibrium



Indeterminate structures

It is easy to find such problems. In the sample problem above, for example, we could have assumed that there is friction between the wall and the top of the ladder. Then there would have been a vertical frictional force acting where the ladder touches the wall, making a total of four unknown forces. With only three equations, we could not have solved this problem.

Consider also an unsymmetrically loaded car. What are the forces—all different—on the four tires? Again, we cannot find them because we have only three independent equations with which to work. Similarly, we can solve an equilibrium problem for a table with three legs but not for one with four legs. Problems like these, in which there are more unknowns than equations, are called **indeterminate.**

To solve such indeterminate equilibrium problems, we must supplement equilibrium equations with some knowledge of *elasticity,* the branch of physics and engineering that describes how real bodies deform when forces are applied to them. The next section provides an introduction to this subject.

12.4 Elastic Properties of Solids

stress: deforming force per unit area, stress = $\frac{F}{A}$

strain: unit deformation, strain = $\frac{\Delta L}{L}$

modulus of elasticity: stress = modulus x strain, Youang's modulus:

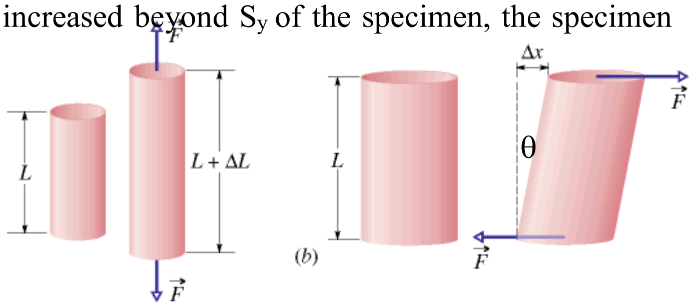
$$Y = \frac{stress}{strain} = \frac{F/A}{\Delta L/L}$$

yield strength: S_y, if the stress is increased beyond S_y of the specimen, the specimen becomes permanently deformed.

Tension & Compression

$\frac{F}{A} = E\frac{\Delta L}{L}$, E: Young's modulus

(a)

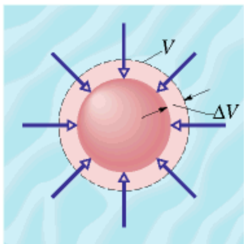


(b)

Shear Stress

$$\frac{F_S}{A} = G\frac{\Delta x}{L}$$

shear strain: $\frac{\Delta x}{L} = \tan\theta$



(c)

shear modulus (torsion modulus): $M_s = \frac{shear\_stress}{shear\_strain} = \frac{F_s/A}{\Delta x/L} = \frac{F_s/A}{\tan\theta}$

Hydraulic Stress

p is the fluid pressure on the object

$$p = -B\frac{\Delta V}{V}$$

Some Elastic Properties of Selected Materials of Engineering Interest				
Material	Density (kg/m^3)	Young's Modulus E (10^9 N/m^2)	Ultimate Strength S_u (10^6 N/m^2)	Yield Strength S_y (10^6 N/m^2)
Steel[a]	7860	200	400	250

Aluminum	2710	70	110	95
Glass	2190	65	50[b]	—
Concrete[c]	2320	30	40[b]	—
Wood[d]	525	13	50[b]	—
Bone	1900	9[b]	170[b]	—
Polystyrene	1050	3	48	—

[a]Structural steel (ASTM-A36).

[b]In compression.

[c]High strength.

[d]Douglas fir.

Sample Example:

A structural steel rod has a radius R of 9.5 mm and a length L of 81 cm. A 62 kN force $\vec{F}$ stretches it along its length. What are the stress on the rod and the elongation and strain of the rod?

$$stress = \frac{F}{A} = \frac{62000}{\pi(9.5\cdot 10^{-3})^2} = 2.2\cdot 10^{8}\, N/m^2$$

$$\frac{\Delta L}{L} = strain = stress/E = \frac{2.2\cdot 10^{8}}{200\cdot 10^{9}} = 1.1\cdot 10^{-3},\quad \Delta L = 81\cdot 1.1\cdot 10^{-3}\, cm = 8.91\cdot 10^{-2}\, cm$$

Example:

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is 1.0 X 10^5 N/m^2 (normal atmosphere pressure). The sphere is lowered into the ocean to a depth where pressure is 2.0 X 10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

$$B = -\frac{P}{\Delta V/V} \rightarrow \Delta V = -\frac{VP}{B},\quad B = 6.2\times 10^{10}\,\text{N/m}^2$$