Lecture 14 Fluids

14.1 Pressure

A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces between molecules and forces exerted by the walls of a container. Both liquids and gases are fluids.

$$P = \frac{F}{A}$$

1 atm = 1.01 x 10^5 Pa = 760 torr = 14.7 PSI (lb/in^2) = 1 Bar = 1000 mBar

A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m.

(a) What does the air in the room weigh when the air pressure is 1.0 atm?

$$W = \rho V g = 1.21 \cdot (3.5 \cdot 4.2 \cdot 2.4) \cdot 9.8 = 418 N$$

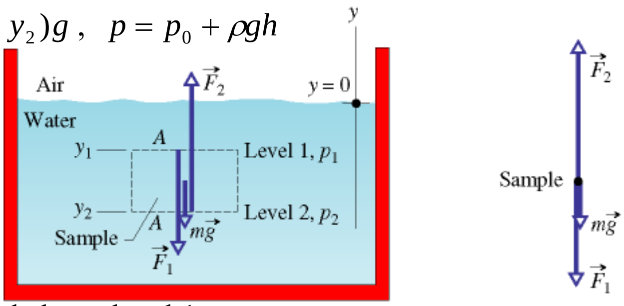
(b) What is the magnitude of the atmosphere's force on the floor of the room?

$$F = pA = 1.01 \cdot 10^5 \cdot (3.5 \cdot 4.2) = 1.5 \cdot 10^6 \, N$$

14.2 Variation of Pressure with Depth

$$F_2 = F_1 + mg\,, \quad p_2 A = p_1 A + \rho A (y_1 - y_2) g\,, \quad p = p_0 + \rho g h$$

in general, the difference between an absolute pressure and an atmosphere pressure is called the gauge pressure



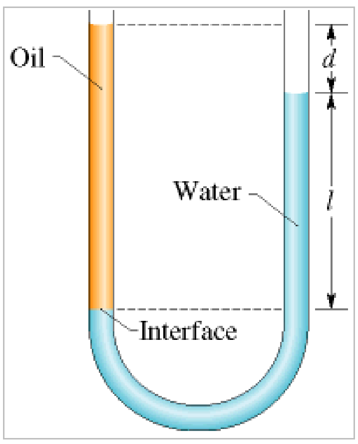
(a) (b)

the atmosphere pressure at a distance d above level 1

$$p = p_0 - \rho_{air} g d$$

Sample Problem:

The U-tube in Figure contains two liquids in static equilibrium: Water of density w (= 998 kg/m3) is in the right arm and oil of unknown density x is in the left. Measurement gives l = 135 mm and d = 12.3 mm. What is the density of the oil?



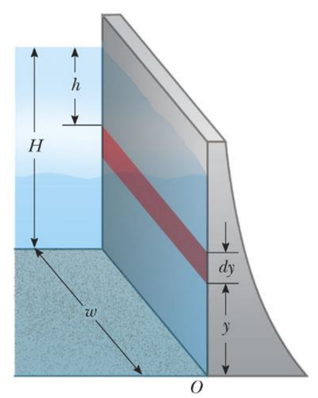
$$\rho_x (d + l) g + 1atm = \rho_w \cdot \mathrm{l}g + 1atm$$

$$\rho_x = \frac{l}{d + l} \rho_w = \frac{135}{135 + 12.3} 998 = 915 kg/m^3$$

Sample Problem:
A novice scuba diver practicing in a swimming pool takes enough air from his tank to fully expand his lungs before abandoning the tank at depth L and swimming to the surface. He ignores instructions and fails to exhale during his ascent. When he reaches the surface, the difference between the external pressure on him and the air pressure in his lungs is 9.3 kPa. From what depth does he start? What potentially lethal danger does he face?

$$9.3\cdot 10^3 = \rho g h = 998\cdot 9.8\cdot h\,,\quad h = \frac{9.3\cdot 10^3}{998\cdot 9.8} = 0.95m$$

Serway/Jewett; Principles of Physics, 3/e
Figure 15.6



Example: The force on a dam
A dam has a water level at a height of H. Assume that the water density is ρ. If the width of the dam is w, please determine the resultant force on the wall of the dam.

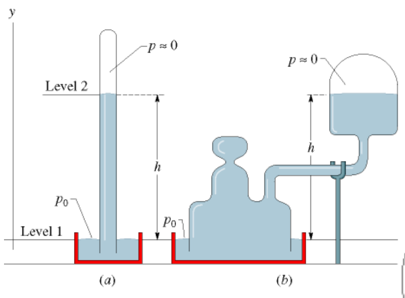
$$P = \rho g h\,,\quad F = \int P dA$$

$$F = \int_0^H \rho g y\cdot w dy = \frac{1}{2}\rho g w H^2$$

14.3 Pressure Measurements

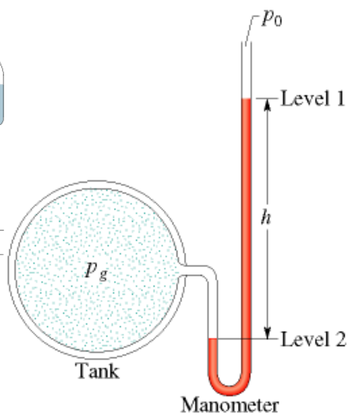
The mercury barometer

$$p_0 = \rho g h$$



The open-tube manometer

$$p_{gauge} = \rho g h\,,\quad p_{real} = p_{gauge} + p_0$$

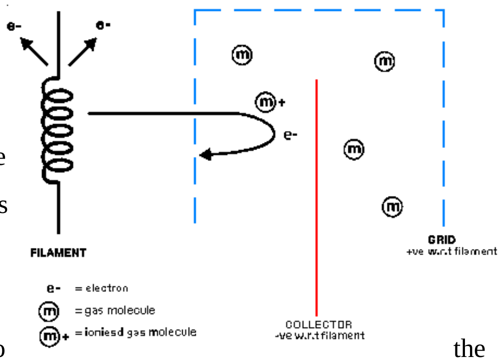


What about vacuum gauge?

The Ion Gauge consists of three distinct parts, the filament, the grid, and the collector. The filament is used for the production of electrons by thermionic emission. A +ve charge on the grid attracts the electrons away from the filament; they circulate around the grid passing through the fine structure many times until eventually they collide with the grid. Gas molecules inside the grid may collide with circulating electrons. The collision can result in the gas molecule being ionised. The collector inside the grid is -ve charged and attracts these +ve charged ions. Likewise they are repelled away from the +ve grid at the same time. The number of ions collected by the collector is directly proportional to the number of molecules inside the vacuum system. By this method, measuring the collected ion current gives a direct reading of the pressure.

The above paragraph is a simplification of what happens. The design of the gauge head effects how efficiently electrons are produced, how long they survive, and how likely they are to collide with a molecule. Combining these factors together gives the gauge a sensitivity. As a general rule, the higher the sensitivity, the more efficient the operation of the gauge.

There are other factors which determine the lowest pressure that a gauge head can measure. One of these limiting factors is the X-Ray Limit. When an electron collides with the grid, there is a probability of a photoelectron being produced. Once generated, there is also a chance that the photoelectron will hit the collector and produce an electron. Unfortunately, the collector does not know the electrical difference between collecting a +ve charge or losing a -ve charge. This means that every time an electron is knocked off the collector, the electronics measure it as receiving a +ve ion instead. This effect is very small and depends on the design of the gauge head. It normally generates a current measured in the picoamp range. At 10^{-10} to 10^{-11} mbar, however, this is also the current produced by the gauge head itself. If pressure is plotted against current, the graph can be seen to tail off as this x-ray current becomes the dominant effect. The x-ray current therefore limits the lowest pressure that the ion gauge can measure.



14,4 Buoyancy Forces and Archimede's Principle

$$B = \rho_f g V$$

When a body is fully or partially submerged in a fluid, a buoyant force b from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight mfg of the fluid that has been displaced by the body.

Floating

When a body floats in a fluid, the magnitude Fb of the buoyant force on the body is equal to the magnitude Fg of the gravitational force on the body.

Apparent weight in a fluid

weight$_{app}$ = weight – F_b

Sample Problem:

What fraction of the volume of an iceberg floating in seawater is visible?

$$\rho_{ice}V = \rho_w V_{under}\,,\quad V_{under} = \frac{\rho_{ice}}{\rho_w}V\,,\quad fraction = \frac{V - V_{under}}{V} = 1 - \frac{\rho_{ice}}{\rho_w}$$

Sample Problem:

A spherical, helium-filled balloon has a radius R of 12.0 m. The balloon, support cables, and basket have a mass m of 196 kg. What maximum load M can the balloon support while it floats at an altitude at which the helium density He is 0.160 kg/m3 and the air density air is 1.25 kg/m3? Assume that the volume of air displaced by the load, support cables, and basket is negligible.

$$196 + M + 0.16\cdot\frac{4\pi}{3}12^3\cdot 9.8 = 1.25\cdot\frac{4\pi}{3}12^3\cdot 9.8$$

$$M = 7694kg$$

14.5 Fluid Dynamics

Ideal fluids in motion:

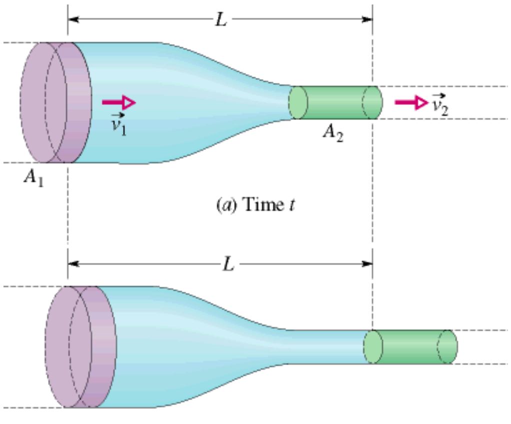
1. **Steady flow**: In steady (or laminar) flow, the velocity of the moving fluid at any fixed point does not change with time, either in magnitude or in direction. The gentle flow of water near the center of a quiet stream is steady; that in a chain of rapids is not. Figure 15-11 shows a transition from steady flow to nonsteady (or turbulent) flow for a rising stream of smoke. The speed of the smoke particles increases as they rise and, at a certain critical speed, the flow changes from steady to nonsteady (that is, from laminar to nonlaminar flow).

2. **Incompressible flow**: We assume, as we have already done for fluids at rest, that our ideal fluid is incompressible; that is, its density has a constant, uniform value.

3. **Nonviscous flow**: Roughly speaking, the viscosity of a fluid is a measure of how resistive the fluid is to flow. For example, thick honey is more resistive to flow than water, and so honey is said to be more viscous than water. Viscosity is the fluid analog of friction between solids; both are mechanisms by which the kinetic energy of moving objects can be transferred to thermal energy. In the absence of friction, a block could glide at constant speed along a horizontal surface. In the same way, an object moving through a nonviscous fluid would experience no viscous drag force—that is, no resistive

force due to viscosity; it could move at constant speed through the fluid. The British scientist Lord Rayleigh noted that in an ideal fluid a ship's propeller would not work but, on the other hand, a ship (once set into motion) would not need a propeller!

4. **Irrotational flow**: Although it need not concern us further, we also assume that the flow is irrotational. To test for this property, let a tiny grain of dust move with the fluid. Although this test body may (or may not) move in a circular path, in irrotational flow the test body will not rotate about an axis through its own center of mass. For a loose analogy, the motion of a Ferris wheel is rotational; that of its passengers is irrotational.



(a) Time t

(b) Time $t + \Delta t$

Streamlines and the continuity equation for fluids

The path taken by a particle of the fluid under steady flow is called a streamline.

$$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$R_v = Av = const$$

$$R_m = \rho R_v = \rho A v$$

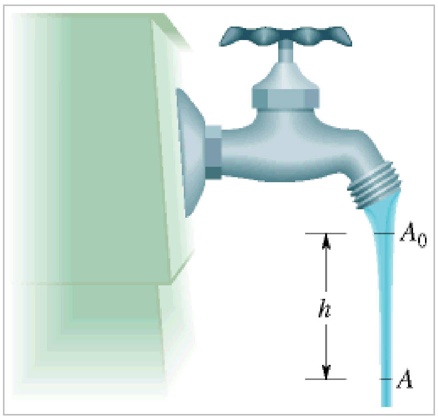
Sample Problem:

The cross-sectional area A_0 of the aorta (the major blood vessel emerging from the heart) of a normal resting person is 3 cm^2, and the speed v_0 of the blood through it is 30 cm/s. A typical capillary (diameter ~6 μm) has a cross-sectional area A of 3 × 10^{-7} cm^2 and a flow speed v of 0.05 cm/s. How many capillaries does such a person have?

$$N = \frac{3 \cdot 30}{3 \cdot 10^{-7} 0.05} = 6 \cdot 10^9$$

Sample Problem:

Figure 15-18 shows how the stream of water emerging from a faucet “necks down” as it falls. The indicated cross-sectional areas are A_0 = 1.2 cm^2 and A = 0.35 cm^2. The two levels are separated by a vertical distance h = 45 mm. What is the volume flow rate from the tap?



$$A_0 v_0 = Av, \quad v^2 = v_0^{\ 2} + 2gh,$$

$$R_v = A_0 v_0 = 34 cm^3 / s$$

14.6 Bernoulli's Equation

$$p_1 + \frac{1}{2}\rho v_1{}^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2{}^2 + \rho g y_2$$

if $y_1 = y_2$

$$p_1 + \frac{1}{2}\rho v_1{}^2 = p_2 + \frac{1}{2}\rho v_2{}^2$$

If the speed of a fluid element increases as it travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.

Proof of Bernoulli's eq:

$$\Delta K = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2 = \frac{1}{2}\rho\Delta V(v_2{}^2 - v_1{}^2)$$

$$\Delta U = \Delta mgh = \rho\Delta V \cdot g(y_2 - y_1)$$

$$W_p = F\Delta x = pA\Delta x = p\Delta V$$

$$W_p = p_1\Delta V - p_2\Delta V$$

$$W_p = \Delta K + \Delta U\ ,\quad p_1\Delta V - p_2\Delta V = \frac{1}{2}\rho\Delta V(v_2{}^2 - v_1{}^2) + \rho\Delta V g(y_2 - y_1)$$

Sample Example:

Ethanol of density = 791 kg/m^3 flows smoothly through a horizontal pipe that tapers in cross-sectional area from A1 = 1.20 $\times 10^{-3}$ m^2 to A2 = A1/2. The pressure difference between the wide and narrow sections of pipe is 4120 Pa. What is the volume flow rate RV of the ethanol?

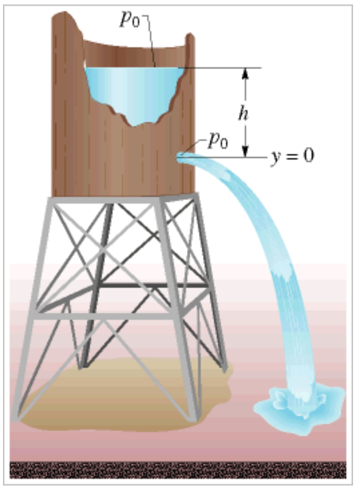
$$A_1 v_1 = A_2 v_2\ ,\quad p_1 + \frac{1}{2}\rho v_1{}^2 = p_2 + \frac{1}{2}\rho v_2{}^2\ ,\quad v_2 = 2v_1\ ,$$

$$\frac{1}{2}\rho({v_2}^2 - {v_1}^2) = 4120$$

$$R_v = A_2 v_2 = 2.24 \cdot 10^{-3} m^3 / s$$

Sample Problem:

In the old West, a desperado fires a bullet into an open water tank (Fig. 15-20), creating a hole a distance h below the water surface. What is the speed v of the water emerging from the hole?



$$\Delta p = \rho g h = \frac{1}{2}\rho(v^2 - 0)\,, \quad v = \sqrt{2gh}$$

14.7 Other applications of fluid dynamics