Lecture 16 Wave Motion

Mechanical waves are waves that disturb and propagate through a medium; the ripple in the water due to the pebble and a sound wave, for which air is the medium, are examples of mechanical waves.

Electromagnetic waves are a special class of waves that do not require a medium in order to propagate, light waves and radio waves are two familiar examples.

16.1 Propagation of a Disturbance

All mechanical waves require (1) some source of disturbance, (2) a medium that can be disturbed, and (3) some physical mechanisms through which particles of the medium can influence one another.

What is phonon?

Transverse waves

Longitudinal waves

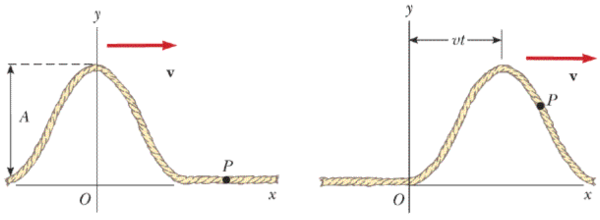
y(x, 0) = f(x)

y(x, t) = y(x - vt, 0)

y(x, t) = f(x – vt); moving to the right

y(x, t) = f(x + vt); moving to the left

Serway/Jewett; Principles of Physics, 3/e
Figure 13.4

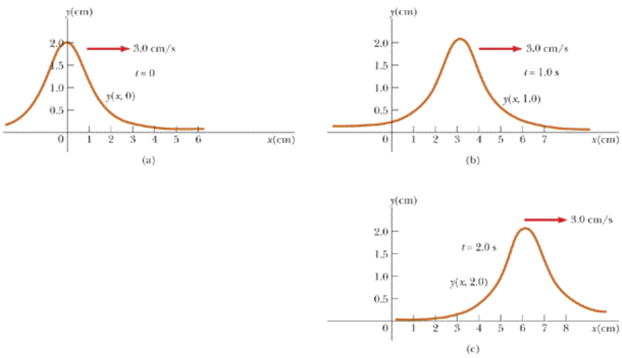


(a) Pulse at $t = 0$ (b) Pulse at time t

Sample Example: A pulse moving to the right

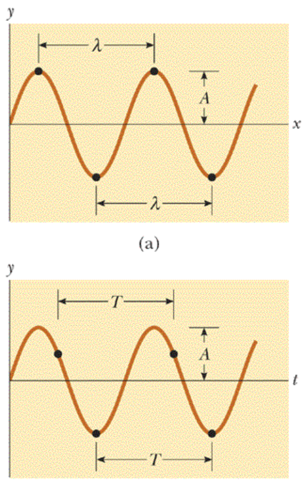
A wave pulse moving to the right along the x-axis is represented by the wave function $y(x,t) = \dfrac{2.0}{(x-3.0t)^2+1}$ where x and y are measured in cm and t is in sec. Let us plot the wave form at t = 0, t = 1, and t = 2 s.

Serway/Jewett; Principles of Physics, 3/e
Figure 13.5



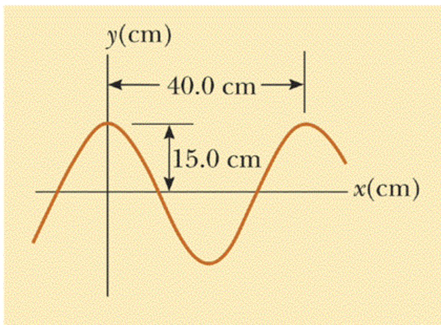
16.2 The Traveling Wave Model

Serway/Jewett; Principles of Physics, 3/e
Figure 13.6



crest: highest displacement
trough: lowest displacement
wave length
frequency
wave speed
Each particle of the string oscillates vertically in the y direction with simple harmonic motion.

Serway/Jewett; Principles of Physics, 3/e
Figure 13.9



at t = 0

$y = A\sin(\frac{2\pi}{\lambda}x)$, if the wave moves to the right with a speed of v

$y = A\sin[\frac{2\pi}{\lambda}(x - vt)]$, $v = \frac{\lambda}{T}$, $y = A\sin[\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t]$

$k = \frac{2\pi}{\lambda}$, $\omega = \frac{2\pi}{T}$, $y = A\sin(kx - \omega t)$, $v = \frac{\omega}{k} = \lambda f$

$kx - \omega t = const$, $k\frac{dx}{dt} - \omega = 0$, $\frac{dx}{dt} = \frac{\omega}{k}$, $v = \frac{\omega}{k} = \lambda f$

if a wave moving in the negative direction of x.

$kx + \omega t = const$, $k\frac{dx}{dt} + \omega = 0$, $\frac{dx}{dt} = -\frac{\omega}{k}$, $v = -\frac{\omega}{k} = -\lambda f$

all traveling waves must be of the form: $y(x,t) = h(kx \pm \omega t)$

Sample Example: A traveling sinusoidal wave

A sinusoidal wave traveling in the positive x direction has an amplitude of 15 cm, a wavelength of 40 cm, and a frequency of 8 Hz. The vertical displacement of the medium at t = 0 and x = 0 is also 15 cm, (a) Find the angular wave number, period, angular frequency, and speed of the wave. (b) Determine the phase constant.

(a) $k = \frac{2\pi}{40}$, $T = \frac{1}{f} = \frac{1}{8}$, $\omega = \frac{2\pi}{T} = 50.3 rad/s$

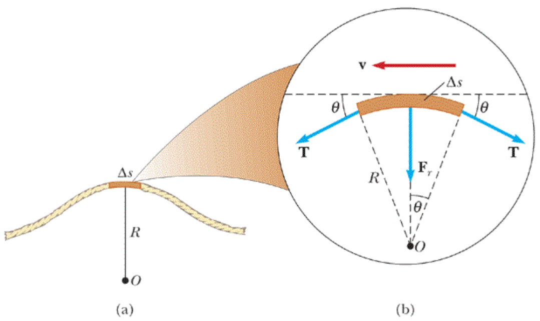
$v = \frac{\omega}{k} = f\lambda = 8 \cdot 40 = 320 cm/s$

(b) $y = 15\sin(\frac{\pi}{20}x - 16\pi t + \phi)$ at t = x = 0, y = 15

$$\sin\phi = 1\,,\quad \phi = \frac{\pi}{2}$$

16.3 The Speed of Waves on Strings

Serway/Jewett; Principles of Physics, 3/e
Figure 13.10



$$F_r = 2T\sin\theta \approx 2T\theta$$

$$m = \mu\Delta s = 2\mu R\theta$$

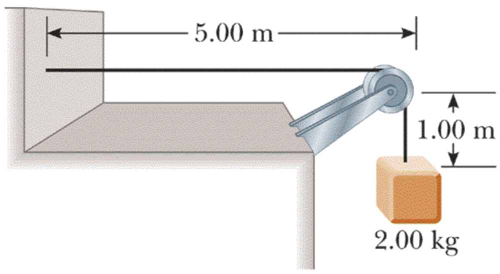
$$F_r = m\frac{v^2}{R}\,,\quad 2T\theta = 2\mu R\theta\frac{v^2}{R}$$

$$T = \mu v^2\,,\quad v = \sqrt{\frac{T}{\mu}}$$

Example: the speed of a pulse on a cord

A uniform cord has a mass of 0.3 kg and a total length of 6 m. Tension is maintained in the cord by suspending an object of mass 2 kg from one end. Find the speed of a pulse on the cord. Assume that the tension is not affected by the mass of the cord.

Serway/Jewett; Principles of Physics, 3/e
Figure 13.11



$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{2\cdot 9.8}{0.3/6}} = 19.8m/s$$

The Wave Equation

$$y = A\sin(kx - \omega t)$$

$$v = \frac{dy}{dt} = -\omega A\cos(kx-\omega t)\,,\quad a = -\omega^2 A\sin(kx-\omega t)$$

$$\frac{dy}{dx} = kA\cos(kx-\omega t)\,,\quad \frac{d^2y}{dx^2} = -k^2A\sin(kx-\omega t)$$

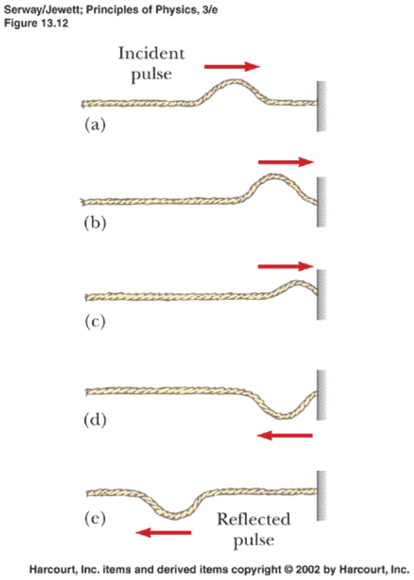
$$\frac{d^2y}{dx^2} = \frac{k^2}{\omega^2}\frac{d^2y}{dt^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

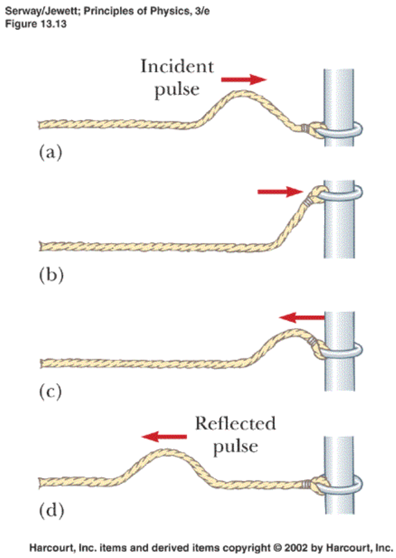
Example: A solution to the linear wave eq

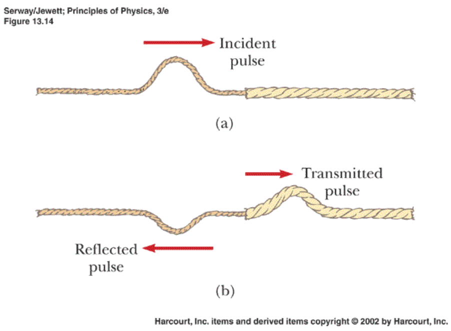
Verify that the wave function presented in previous example is a solution to the linear wave eq.

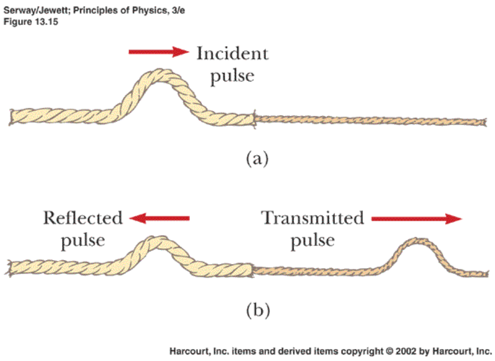
$$y = \frac{2}{(x-3t)^2+1}\,,\quad \frac{d^2y}{dx^2} = \frac{1}{9}\frac{d^2y}{dt^2}$$

16.4 Reflection and Transmission









$$y_l(x,t) = A\cos(k_l x - \omega t),\quad v_l = \sqrt{\frac{T}{\mu_l}} = \frac{\omega}{k_l},$$

$$y_l'(x,t) = B\cos(-k_l x - \omega t)$$

$$y_r(x,t) = C\cos(k_r x - \omega t),\quad v_r = \sqrt{\frac{T}{\mu_r}} = \frac{\omega}{k_r}$$

$$x = 0,\quad y_l + y_l' = y_r,\ ==>\quad A + B = C$$

$$\frac{d}{dx}y_l + \frac{d}{dx}y_l' = \frac{d}{dx}y_r,\ ==>\quad k_l(A - B) = k_r C$$

$B = \dfrac{k_l - k_r}{k_l + k_r}A,\quad C = \dfrac{2k_l}{k_l + k_r}A$, light string -> small μ -> small k

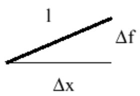
16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings

$$\Delta K = \frac{1}{2}\Delta m v^2 = \frac{1}{2}\mu \Delta x v^2, \quad dK = \frac{1}{2}\mu v^2 dx = \frac{1}{2}\mu\omega^2 A^2 \sin^2(kx-\omega t)dx$$

at t = 0, average kinetic energy in a period of wave length

$$K_\lambda = \frac{1}{2}\mu\omega^2 A^2 \int_0^\lambda \sin^2(kx)dx, \quad \int_0^\lambda \sin^2(kx)dx = \int_0^\lambda \frac{1-\cos(2kx)}{2}dx = \frac{\lambda}{2}$$

$$K_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$



the change is length from Δx to l is: $\Delta x\sqrt{1+(\frac{dy}{dx})^2} - \Delta x \approx \frac{1}{2}(\frac{dy}{dx})^2 \Delta x$,

$$\Delta U = T \cdot (l - \Delta x) = \frac{1}{2}T(\frac{dy}{dx})^2 \Delta x$$

$$\Delta U = \frac{1}{2}T(\frac{dy}{dx})^2 \Delta x = \frac{1}{2}\mu v^2 (\frac{dy}{dx})^2 \Delta x = \frac{1}{2}\mu(\frac{\omega}{k})^2(\frac{dy}{dx})^2 \Delta x,$$

$$dU = \frac{1}{2}\mu\omega^2 A^2 \sin^2(kx-\omega t)dx$$

at t = 0, $U_\lambda = \frac{1}{2}\mu\omega^2 A^2 \int_0^\lambda \sin^2(kx)dx, \quad \int_0^\lambda \sin^2(kx)dx = \int_0^\lambda \frac{1-\cos(2kx)}{2}dx = \frac{\lambda}{2}$

$$K_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda, \quad P = \frac{E_\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 v$$

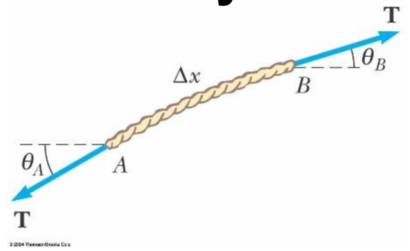
Sample Example:

A string with linear mass density 5 x 10^{-2} kg/m is under a tension of 80 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60 Hz and an amplitude of 6 cm?

$$A = 0.06m, \quad f = 60, \quad \mu = 0.05kg/m, \quad v = \sqrt{\frac{80}{0.05}} = 40$$

$$P = \frac{1}{2}0.05 \cdot 60^2 \cdot 0.06^2 \cdot 40 = 512W$$

16.6 The Linear Wave Equation – Physical Model



$$\sum F_y = T\sin\theta_B - T\sin\theta_A \approx T\tan\theta_B - T\tan\theta_A$$

$$\sum F_y = T\left(\frac{\partial y}{\partial x}\right)_B - T\left(\frac{\partial y}{\partial x}\right)_A$$

$$\sum F_y = ma_y = m\frac{\partial^2 y}{\partial t^2} = \mu\Delta x\frac{\partial^2 y}{\partial t^2} = T\left(\frac{\partial y}{\partial x}\right)_B - T\left(\frac{\partial y}{\partial x}\right)_A = T\frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)\Delta x$$

$\mu\Delta x\frac{\partial^2 y}{\partial t^2} = T\frac{\partial^2 y}{\partial x^2}\Delta x$ the sinusoidal wave function is $y(x,t) = A\sin(kx - \omega t)$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(kx - \omega t),\quad \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx - \omega t)$$

$\mu\omega^2 = Tk^2$ -> $v = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$ -> $\frac{\partial^2 y}{\partial x^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$