Chapter 23 Gauss's Law

23.1 Electric Flux

For a surface perpendicular to $\vec{E}$, the electric flux is $\phi = EA$.

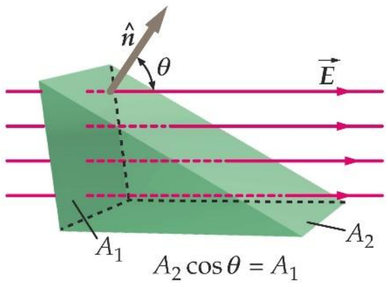
Units: N*m^2/C

$\Phi = EA$

The mathematical quantity that corresponds to the number of field lines penetrating a surface is called the electric flux ϕ.

$A_1 = A_2 \cos\theta$

$\phi = EA_1 = EA_2 \cos\theta$



$$\phi = \vec{E} \cdot \hat{n}A = EA\cos\theta$$

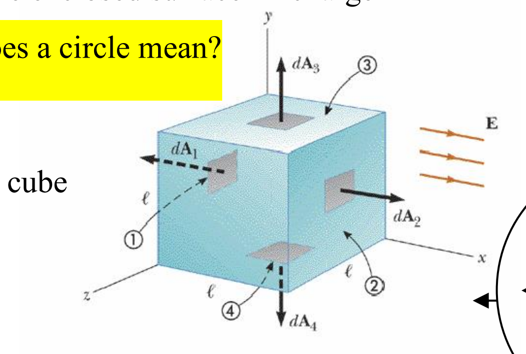
$$\phi = \int_S \vec{E} \cdot \hat{n}dA$$

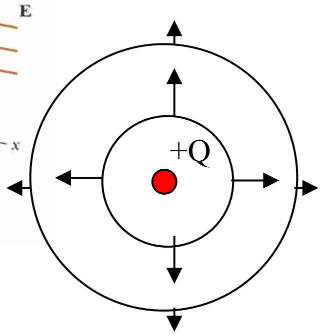
The electric flux out of the enclosed surface → charge

$\phi = \oint_S \vec{E} \cdot \hat{n}dA$ -- What does a circle mean?

Example: Flux through a cube

$\Phi = -El^2 + El^2 = 0$



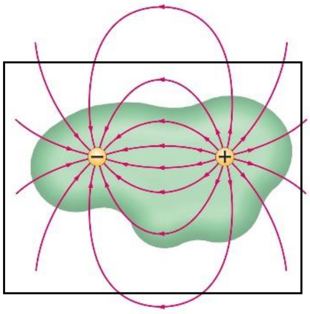
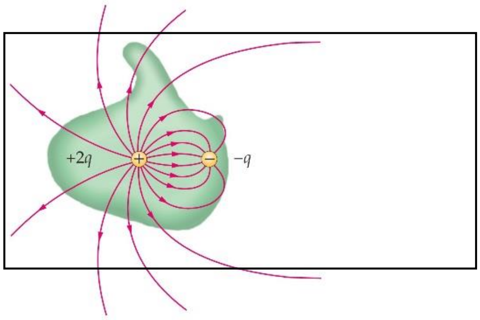


Total flux of the enclosed surface do not change --> charge

Do not mix the vector field representation with the electric field line representation.

23.2 Gauss's Law

Electric fields from symmetrical charge distribution can be easily calculated using Gauss's law.

The net number of lines out of any surface enclosing the charges is proportional to the net charge enclosed by the surface.

Quantitative Statement of Gauss's Law

$$\phi = EA = k\frac{q}{r^2}4\pi r^2 = 4\pi kq = \frac{q}{\varepsilon_0}$$

$$\phi_{net} = \oint \vec{E}\cdot d\vec{A} = 4\pi kq = \frac{q}{\varepsilon_0}$$

System containing multiple charges:

$\phi_{net} = \int_S E_n dA = 4\pi kQ_{enc}$ → Gauss's law

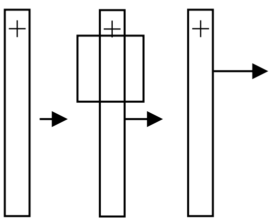
ε_0: permittivity of free space, $k = \frac{1}{4\pi\varepsilon_0}$ & $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ & $\phi_{net} = \oint \vec{E}\cdot d\vec{a} = \frac{Q_{net}}{\varepsilon_0}$

$$\phi_{net} = \oint \vec{E}\cdot d\vec{A} = \int \nabla\cdot\vec{E}dV = 4\pi kq = 4\pi k\int \rho dV$$

$\left(\nabla\cdot\vec{E}\right)_x = \frac{\Delta E_x}{\Delta x} = \frac{\partial E_x}{\partial x}$ Divergence: change of electric flux per unit volume

$$\left(\Delta\Phi_E\right)_x = E_x(x+\Delta x)dydz - E_x(x)dydz = \frac{\partial E_x}{\partial x}dxdydz$$

→ $\left(\nabla\cdot\vec{E}\right)_x = \frac{\left(\Delta\Phi_E\right)_x}{dxdydz} = \frac{\partial E_x}{\partial x}$ → $\nabla\cdot\vec{E} =$



$1\hat{i}, 2\hat{i}, 3\hat{i}$ → $\nabla \cdot \vec{E} = \frac{2-1}{1} = 1$ → 1,0,1,0,1,0,1,0 → charge distribution →

$\nabla \cdot \vec{E} = 4\pi k \rho$

Example: An electric field is $\vec{E} = (200\ N/C)\hat{i}$ in the region $x > 0$ and

$\vec{E} = -(200\ N/C)\hat{i}$ in the region $x < 0$. An imaginary soup-can shaped surface of length 20 cm and radius R = 5 cm has its center at the origin and its axis along the x axis, so that one end is at x = +10 cm and the other is at x = -10 cm. (a) What is the net outward flux through the entire closed surface? (b) What is the net charge inside the closed surface?

(a) $\phi = (200\ N/C)\hat{i} \cdot \left(\pi (0.05\ m)^2\right)\hat{i} + (-200\ N/C)\hat{i} \cdot \left(\pi (0.05\ m)^2\right)\left(-\hat{i}\right) = 3.14$ (N m^2/C)

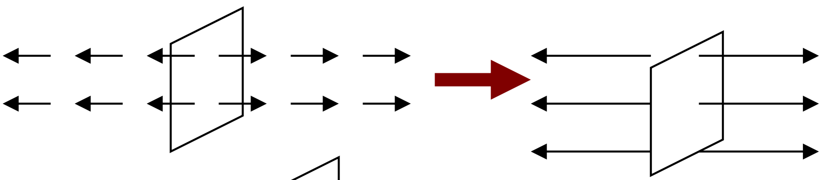
(b) $Q = \varepsilon_0 \phi = 27.8$ (pC)

23.3 Application of Gauss's Law to Various Charge Distribution

Symmetry → Use integration
(choose spherical, cylindrical, or Cartesian coordinate systems)
→ Use Gauss's law will be easier to solve the problem.

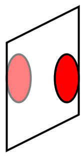
Gauss's Law: $\oint \vec{E} \cdot d\vec{a} = \phi_{net} = 4\pi k Q_{enclosed}$

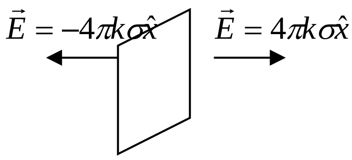
Plane Symmetry

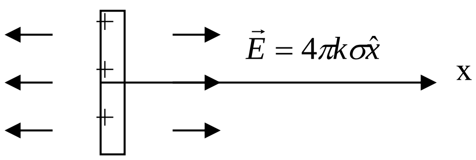


$AE_{\leftarrow} + AE_{\rightarrow} = 2AE = 4\pi k Q_{enc} = 4\pi k A \sigma$

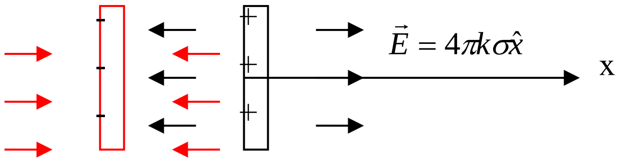
$\vec{E} = 4\pi k \sigma \hat{n}$

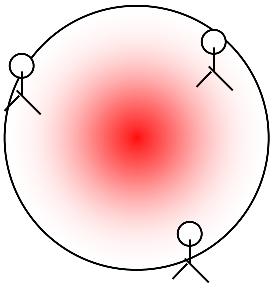






Example: Electric Fields Due to Two Infinite Planes



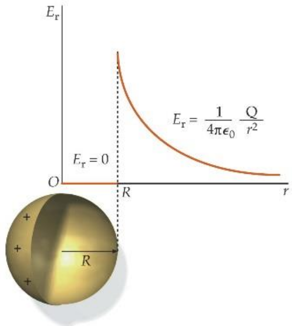


Spherical Symmetry

$$\phi_{net} = \oint \vec{E}\cdot d\vec{a} = (E\hat{r})\cdot(4\pi r^2\hat{n}) = 4\pi r^2 E = 4\pi kQ = \frac{Q}{\varepsilon_0}$$

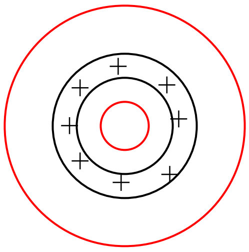
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

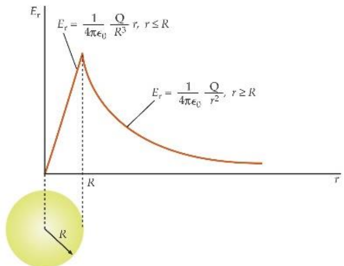
$\vec{E}$ Due to a Thin Spherical Shell of Charge



$r > R$ --> $4\pi r^2 E = \dfrac{Q}{\varepsilon_0}$

$r < R$ --> $4\pi r^2 E = \dfrac{0}{\varepsilon_0}$





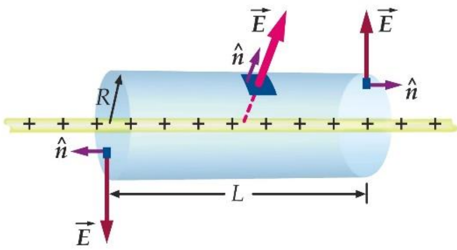
Example: Find the electric field outside and inside a uniformly charged solid sphere of radius R carrying a total charge Q.

Describe Divergence in Spherical Coordinate System

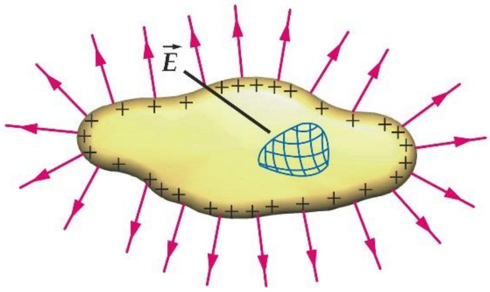
Cylindrical Symmetry

$$E2\pi RL = \frac{Q}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

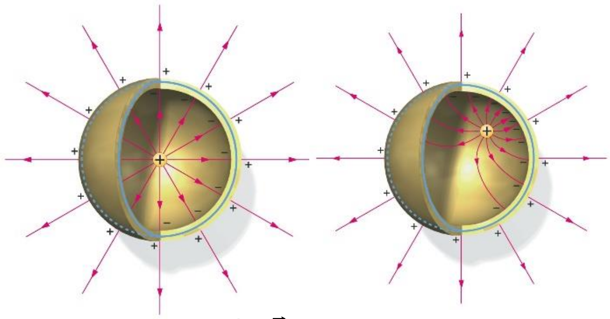
$$\vec{E} = \frac{\lambda}{2\pi R\varepsilon_0}\hat{r}$$



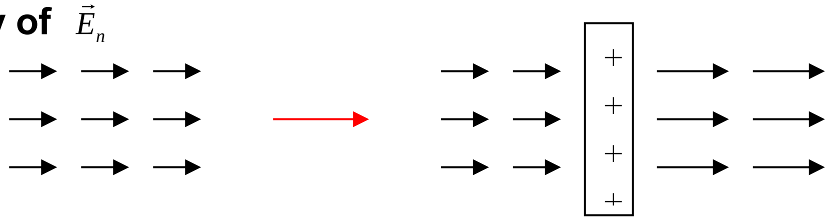
23.4 Conductors in Electrostatic Equilibrium



1. Because the charges are arranged to have zero electric field inside the conductor. (If the field is not zero, the charges will move without stop.)
2. The charge resides on surfaces of a conductor.
3. The electric field is perpendicular to the conductor surface. ΔE across the boundary surface is equal to $\frac{\sigma}{\varepsilon_0}$.
4. You can also use Gauss's law to derive the same result.



Discontinuity of $\vec{E}_n$



Charge --> The Source to increase or decrease electric fields

$$\vec{E} = \vec{E}' + \vec{E}_{disk}$$

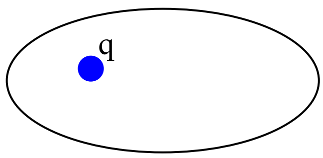
$$\vec{E}_{n+} = \vec{E}' + \frac{\sigma}{2\varepsilon_0}\hat{i} \quad \& \quad \vec{E}_{n-} = \vec{E}' - \frac{\sigma}{2\varepsilon_0}\hat{i} \quad \text{-->} \quad \Delta\vec{E} = \vec{E}_{n+} - \vec{E}_{n-} = \frac{\sigma}{\varepsilon_0}\hat{i}$$

Derivation of Gauss's Law From Coulomb's Law

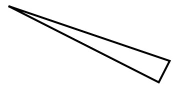
$$\Delta\Omega = \frac{\Delta A}{r^2}$$

The total solid angle of the spherically symmetrical space is $\frac{4\pi r^2}{r^2} = 4\pi$.

$$\Delta\theta = \frac{\Delta s}{r}$$



$$\Delta\Omega = \frac{\Delta A(\hat{n} \cdot \hat{r})}{r^2} = \frac{\Delta A \cos\theta}{r^2}$$



$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

The outgoing electric flux through a small area ΔA is $\Delta\phi = \vec{E} \cdot \hat{n}\Delta A$.

$$\Delta\phi = \frac{kq}{r^2}(\hat{r} \cdot \hat{n})\Delta A = kq\Delta\Omega \rightarrow \phi_{net} = \int d\phi = \oint(\vec{E} \cdot \hat{n})da = kq\oint d\Omega = 4\pi kq$$