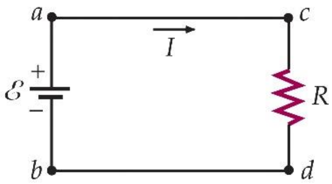
Chapter 27 Direct Current Circuits

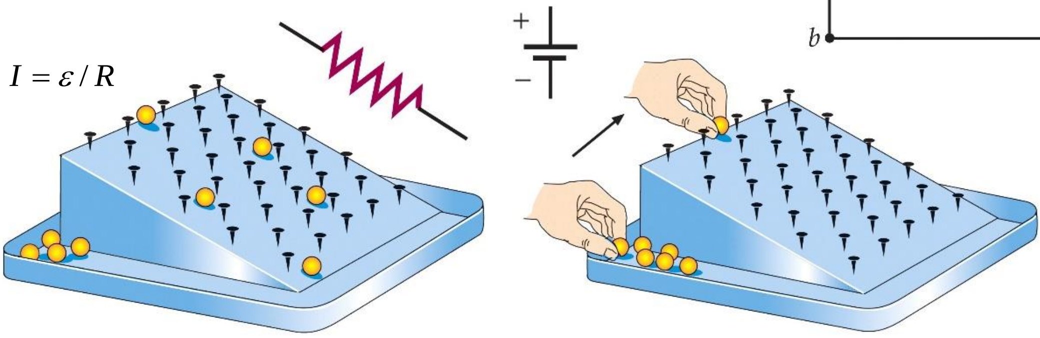
Here we analyze simple electric circuits that contains batteries, resistors, and capacitors in various combinations.

27.1 Electromotive Force

A device that supplies electrical energy to a circuit is called a source of emf, ε .



$I = \varepsilon / R$

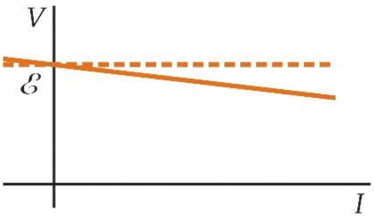


$$P = \frac{\Delta Q \varepsilon}{\Delta t} = I\varepsilon$$

Because a real battery is made of matter, there is a resistance to the flow of charge within the battery: internal resistance r .

Therefore, the terminal voltage of the battery is $\Delta V = V_A - V_B = \varepsilon - Ir$.

ε is equivalent to the open-circuit voltage.



$IR = V_{\text{terminal}} = \varepsilon - Ir$ --> $I = \dfrac{\varepsilon}{R+r}$

$$V_{\text{terminal}} = \varepsilon - \frac{r\varepsilon}{R+r} = \frac{R}{R+r}\varepsilon$$

An increase of I means a decrease of R so V_{terminal} will be reduced.

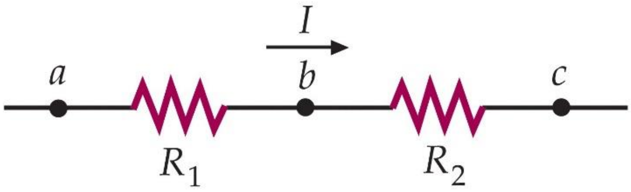
1 Ah = (1 C / s) (3600 s) = 3600 C

Example: For a battery of given emf ε and internal resistance *r*, what value of external resistance *R* should be placed across the terminals to obtain the maximum power delivered to the resistor?

$P = I^2 R = \left(\dfrac{\varepsilon}{R+r}\right)^2 R$, $dP/dR = 0$ --> $R = r$

27.2 Resistors in Series and Parallel

Resistors in Series

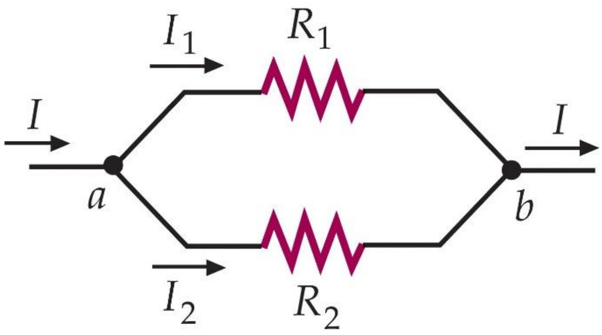


$V = V_1 + V_2$

$I = I_1 = I_2$

$R = \frac{V}{I} = \frac{V_1 + V_2}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} = R_1 + R_2$

$R_{eq} = R_1 + R_2 + R_3 + \ldots$



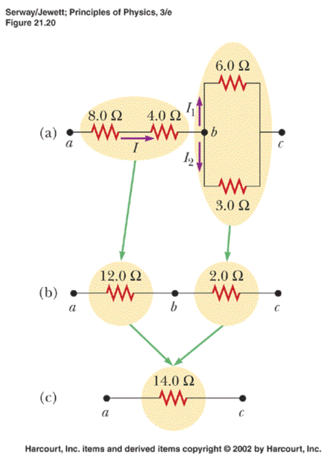
Resistors in Parallel

$V = V_1 = V_2$

$I = I_1 + I_2$ -> $\frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

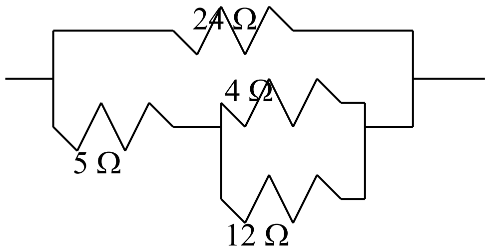


Example: Find the equivalent resistance

Example: You are making a snack for some friends to help you get ready for a full night of studying. You decide that coffee, toast, and popcorn would be a good start. You start the toaster and get some popcorn going in the microwave. Since your apartment is in an older building, you know you have problems with the fuse blowing when you turn too many things on. Should you start the coffeemaker? You look on the appliance and find that the toaster has a rating of 900 W, the microwave is rated at 1200 W, and the coffeemaker is rated at 600 W. Past experience with replacing fuses has shown that your house has 20-A fuses.

$I_{toaster} = \frac{900}{120} = 7.5\,(\mathrm{A})$ & $I_{m-wave} = \frac{1200}{120} = 10\,(\mathrm{A})$ & $I_{c-maker} = \frac{600}{120} = 5\,(\mathrm{A})$

Example: Find the equivalent resistance of the combination of resistors shown in figure.



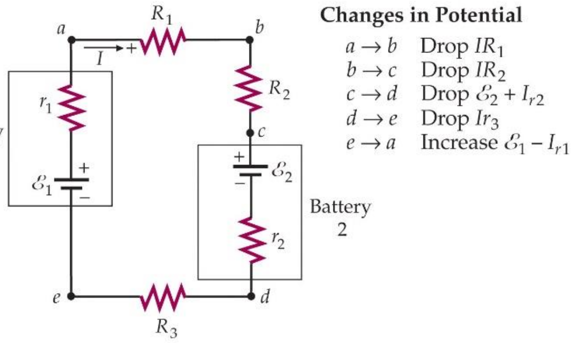
27.3 Kirchhoff's Rules

Kirchhoff's rules:

1. When any closed-circuit loop is traversed, the algebraic sum of the changes in potential must equal zero. (loop rule: $\vec{E}$ is conservative: $\oint \vec{E} \cdot d\vec{l} = 0$)

2. At any junction (branch point) in a circuit where the current can divide, the sum of the currents into the junction must equal the sum of the currents out of the junction. (junction rule: conservation of charge: $I = I_1 + I_2$)

Single-Loop Circuit



Battery: from – to + means positive emf

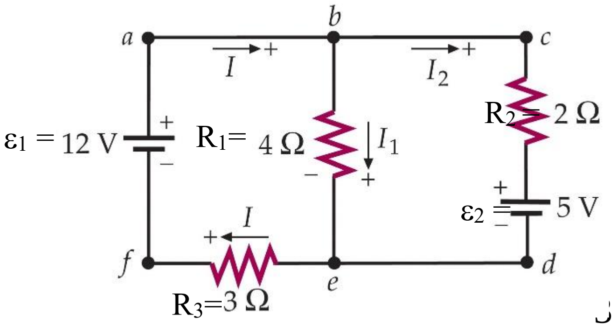
Resistance: consume electric potential

$$\varepsilon_1 - Ir_1 - IR_1 - IR_2 - \varepsilon_2 - Ir_2 - IR_3 = 0$$

$$I = \frac{\varepsilon_1 - \varepsilon_2}{r_1 + R_1 + R_2 + r_2 + R_3}$$

Example: A fully charged car battery is to be connected by jumper cables to a discharged car battery in order to charge it. (a) To which terminal of the discharged battery should the positive terminal of charged battery be connected? (b) Assume that the charged battery has an emf of 12 V and the discharged battery has an emf of 11 V, that the internal resistances of the batteries are $r_1 = r_2 = 0.02\Omega$, and that the resistance of the jumper cables is 0.01 Ω. What will the charging current be? (c) What will the current be if the batteries are connected incorrectly?

Multi-Loop Circuit



Current: forward direction: positive;

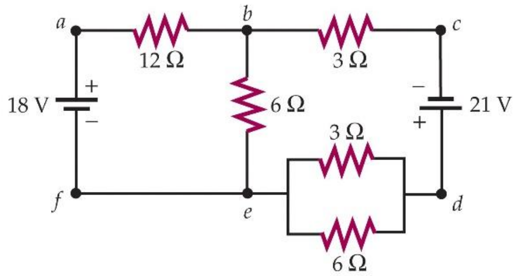
backward: negative

$I = I_1 + I_2$

$\varepsilon_1 - I_1 R_1 - I R_3 = 0$

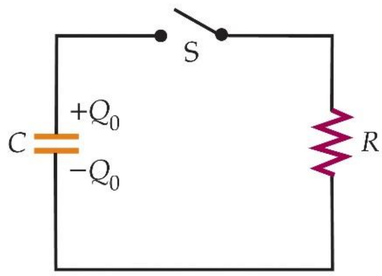
$\varepsilon_1 - I_2 R_2 - \varepsilon_2 - I R_3 = 0$ (or $-I_2 R_2 - \varepsilon_2 - (-I_1) R_1 = 0$)

Example:



27.4 RC Circuits

Discharging a Capacitor



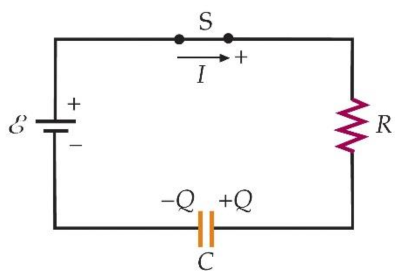
Diff. Eq.

$-\frac{Q}{C} - IR = 0$ --> $\frac{Q}{C} + R\frac{dQ}{dt} = 0$

$\frac{dQ}{dt} = -\frac{Q}{RC}$ --> $\frac{dQ}{Q} = -\frac{dt}{RC}$

$[\ln Q]_{Q_0}^{Q} = -\frac{t}{RC}$ --> $Q = Q_0 e^{-\frac{t}{RC}}$ & $I = \frac{Q_0}{RC} e^{-\frac{t}{RC}}$, where $\tau = RC$ called the **time constant.**

Charging a Capacitor



$\varepsilon - IR - \frac{Q}{C} = 0$ --> $R\frac{dQ}{dt} = \varepsilon - \frac{Q}{C} = \frac{C\varepsilon - Q}{C}$

$\frac{d(C\varepsilon - Q)}{C\varepsilon - Q} = -\frac{dt}{RC}$ --> $[\ln(C\varepsilon - Q)]_0^Q = -\frac{t}{RC}$ --> $C\varepsilon - Q = C\varepsilon e^{-\frac{t}{RC}}$

$$Q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) \text{ --> } I = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

Energy Conservation in Charging a Capacitor

A total charge $Q_f = VC$ flow through the battery and the battery does work

$W = Q_f V = V^2 C$.

The energy stored in the capacitor is $\int V dQ = \int_0^{Q_f} \frac{Q}{C} dQ = \frac{Q_f^2}{2C} = \frac{1}{2} V^2 C$.

The other half of energy is dissipated by the resistance *R* of the circuit.

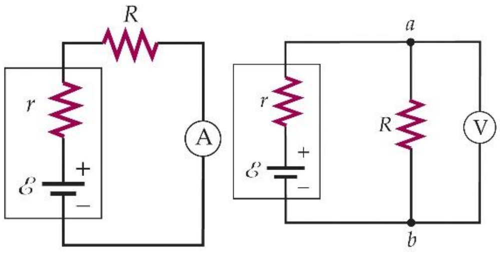
$$I = \frac{V}{R} e^{-\frac{t}{RC}}$$

$$W = \int_0^\infty I^2 R dt = \frac{V^2}{R} \int_0^\infty e^{-\frac{2t}{RC}} dt = \frac{C}{2} V^2$$

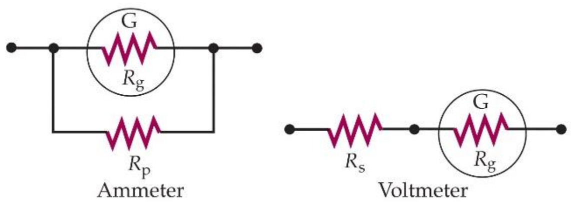
27.5 Electrical Meters

Ammeters, Voltmeters, and Ohmmeters

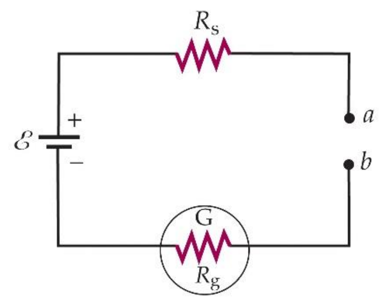
Connection Methods for ammeters and voltmeters:



Shunt resistors of ammeters ($R_s << R_g$) and voltmeters ($R_s >> R_g$):



A galvanometer is a device that detects a small current passing through it.
Ohmmeter: a battery connected with a galvanometer, when a and b are shorted, the galvanometer gives a full-scale deflection.



27.6 Household Wiring and Electrical Safety