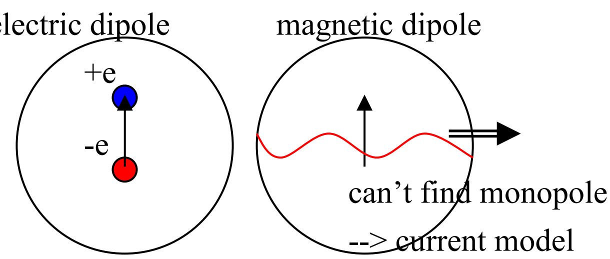
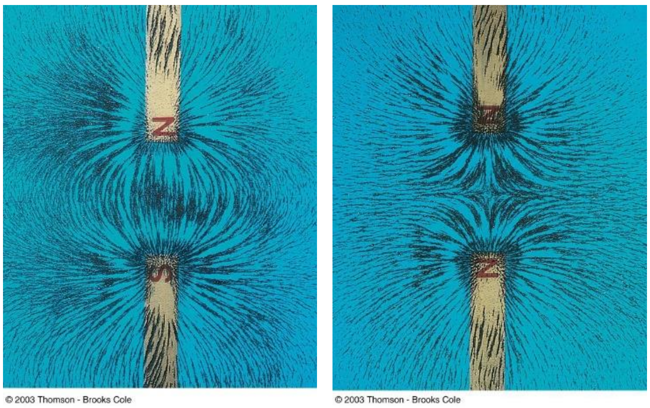
Chapter 28 Magnetic Fields



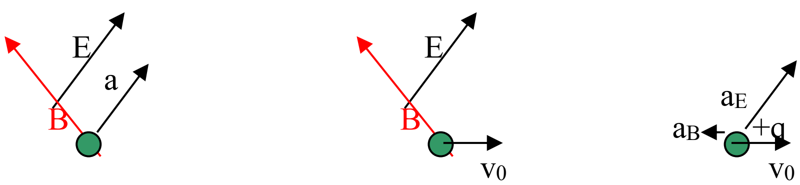
Magnetic poles always occur in pairs.
Thus far, there is no conclusive evidence that an isolated magnetic monopole exists.

28.1 Magnetic Fields and Forces



We can define a magnetic field $\vec{B}$ at some point in space in terms of the magnetic force $\vec{F}_B$.

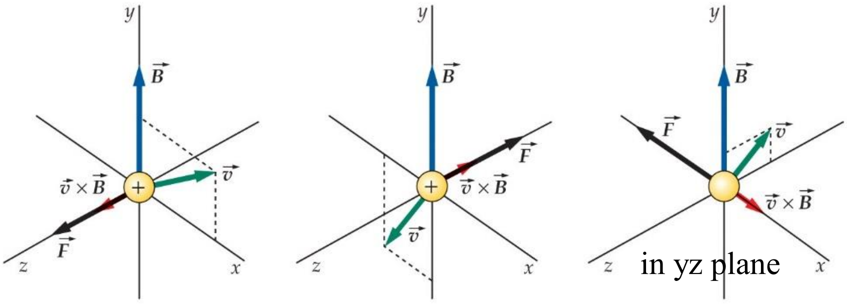
Lorentz force law: When a charge q moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$, the magnetic force $\vec{F}$ on the charge is $\vec{F} = q\vec{v} \times \vec{B}$.



1. The static charge doesn't response to the magnetic field. Only a moving charge will feel a magnetic-field induced force.
2. The magnetic force is perpendicular to the plane spanned by $\vec{v}$ and $\vec{B}$.
3. Unit of the magnetic field: tesla (T) = 1 N / (C (m/s)), 1 T = 10^4 Gauss
4. The magnetic field on the earth is 0.44 G. The maximum of static magnetic field

at laboratory is ~ 10 tesla. The permanent magnet produces the magnetic field of ~1 T.

5. What's a permanent magnet? Is there any difference between fields at laboratory and from a permanent magnet?



Magnetic Field Lines

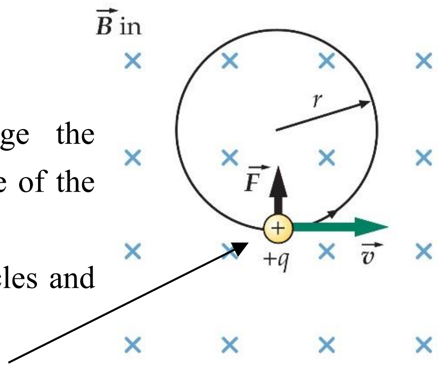
N is like a positive charge and S is like a negative charge.
Inside the magnet the field line direction is like that in an electric dipole.

TABLE 29.1

Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	30
Strong conventional laboratory magnet	2
Medical MRI unit	1.5
Bar magnet	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	0.5×10^{-4}
Inside human brain (due to nerve impulses)	10^{-13}

28.2 Motion of a Charged Particle in a Uniform Magnetic Field



1. The magnetic force (Lorentz force) change the direction of the velocity but not the magnitude of the velocity.
2. Magnetic fields do not work on charged particles and

do not change their kinetic energy.

When the initial velocity $\vec{v}$ is perpendicular to the magnetic field, the particle will do circular motion.

$$F = ma = m\frac{v^2}{r} = F_{Lorentz} = qvB \quad (q\vec{v}\times\vec{B} = qvB\sin(90^o) = qvB)$$

$r = \frac{mv}{qB}$ (The radius of the particle's circular motion depends linearly on its velocity and its mass, but inversely on its charge and the magnetic field. --> You can use the relation to measure the velocity of charged particles.)

The period of the cyclotron motion is $T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$.

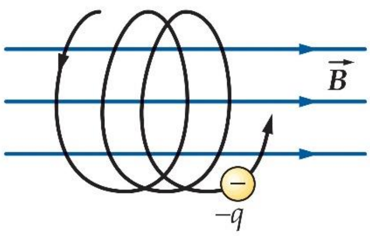
The cyclotron frequency is $f = \frac{1}{T} = \frac{qB}{2\pi m}$, and the angular speed is $\omega = 2\pi f = \frac{qB}{m}$.

Example: A proton of mass $m = 1.67\times10^{-27}$ kg and charges $q = e = 1.6\times10^{-19}$ C moves in a circle of radius $r = 21$ cm perpendicular to a magnetic field of $B = 4000$ G. Find (a) the period of the motion and (b) the speed of the proton.

$$qvB = m\frac{v^2}{r} \text{ --> } v = \frac{qBr}{m} = \frac{(1.6\times10^{-19}C)(0.4T)(0.21m)}{1.67\times10^{-27}kg} = 8.05\times10^6 m/s$$

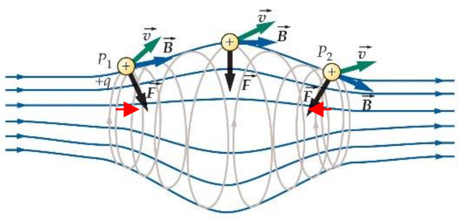
$$T = \frac{2\pi r}{v} = \frac{2\pi(0.21m)}{8.05\times10^6 m/s} = 1.64\times10^{-7}s$$

If the velocity is not perpendicular to the magnetic field, the motions can be separated to two independent parts: along the field direction and in the plane perpendicular to the field.



Motion in nonuniform magnetic field:

The magnetic field is weak at the center and strong at both ends. The particles spiral around the field lines and become trapped, oscillating back and forth between the two ends. Why?



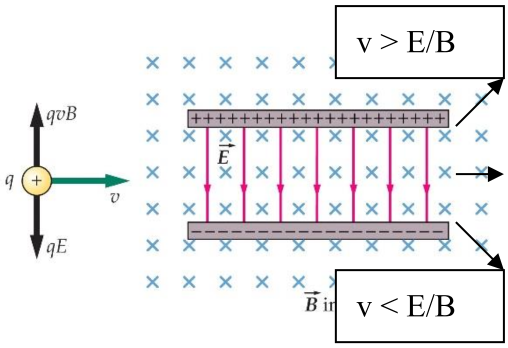
<u>28.3 Applications Involving Charged</u>

Particles Moving in a Magnetic Field

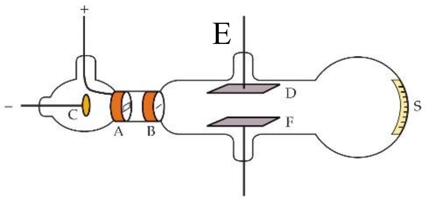
The Velocity Selector

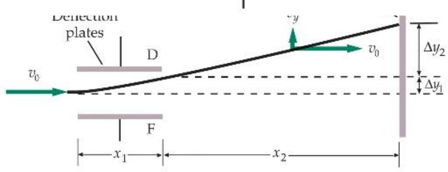
Balance of electric and magnetic force is used for velocity selection.

$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ When the directions of $q\vec{E}$ and $q\vec{v} \times \vec{B}$ are opposite, the particle undergoes linear motion with the velocity v having the relation $qE = qvB$.



Thomson's Measurement of q/m for Electrons





The experiments were performed by J. J. Thomson in 1897. --> The rays of a cathode-ray tube can be deflected by electric and magnetic fields.

Thomson's method: using electric field to bend the cathode-ray and to measure the charge-to-mass ratio.

Accelerating region (region 1):

$$a_y = \frac{qE}{m} \quad \& \quad t_1 = \frac{x_1}{v_0} \quad \text{-->} \quad y_1 = \frac{1}{2}at_1^2 = \frac{1}{2}\frac{qE}{m}\frac{x_1^2}{v_0^2}$$

Constant velocity region (region 2):

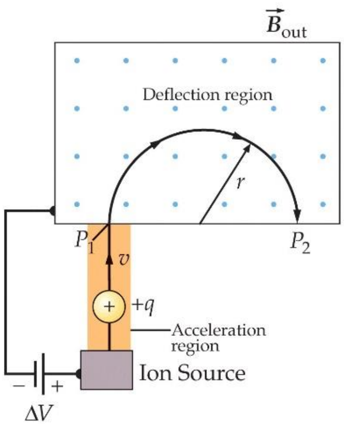
$$v_{y2} = a_y t_1 = \frac{qE}{m}\frac{x_1}{v_0} \quad \& \quad t_2 = \frac{x_2}{v_0} \quad \text{-->} \quad y_2 = \frac{qE}{m}\frac{x_1}{v_0}\frac{x_2}{v_0}$$

$$y = \frac{1}{2}\frac{qE}{m}\frac{x_1^2}{v_0^2} + \frac{qE}{m}\frac{x_1}{v_0}\frac{x_2}{v_0} = \frac{q}{m}E\left(\frac{1}{2}\frac{x_1^2}{v_0^2} + \frac{x_1}{v_0}\frac{x_2}{v_0}\right)$$

The Mass Spectrometer

Electric field accelerating region:

$$\frac{1}{2}mv^2 = qV$$



The magnetic field bending region:

$$qvB = m\frac{v^2}{r} \quad \text{-->} \quad v = \frac{qBr}{m}$$

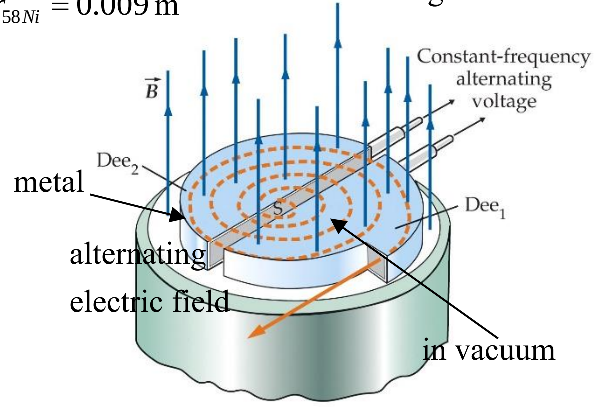
$$\text{-->} \quad \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = qV \quad \text{-->} \quad \frac{m}{q} = \frac{B^2r^2}{2V}$$

If the magnetic field and the accelerating voltage are constant, the radius r will depend on the mass-to-charge ratio. ($r \propto \sqrt{q/m}$)

Example: A ^{58}Ni ion of charge +e and mass 9.62×10^{-26} kg is accelerated through a potential drop of 3 kV and deflected in a magnetic field of 0.12 T. (a) Find the radius of curvature of the orbit of the ion. (b) Find the difference in the radii of curvature of ^{58}Ni ions and ^{60}Ni ions.

(a) $r = \sqrt{\frac{2mV}{qB^2}} = \sqrt{\frac{2(9.62\times10^{-26})(3000)}{(1.602\times10^{-19})(0.12)^2}} = 0.501\,\text{m}$

(b) $r_{60Ni} = r\sqrt{\frac{60}{58}} = 0.510\,\text{m}$ --> $r_{60Ni} - r_{58Ni} = 0.009\,\text{m}$



The Cyclotron

A charged-particle accelerator:

$$qvB = m\frac{v^2}{r} \quad \text{-->} \quad v = \frac{qBr}{m}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \quad \text{-->} \quad \omega = \frac{2\pi r}{T} = \frac{qB}{m}$$

The kinetic energy of a charged particle leaving the cyclotron is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2}{2m}r^2$$

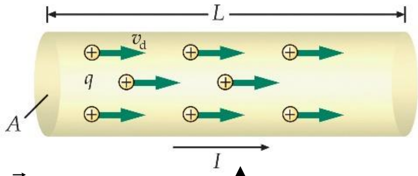
Example: A cyclotron for accelerating protons has a magnetic field of 1.5 T and a maximum radius of 0.5 m. (a) What's the cyclotron frequency? (b) What's the kinetic energy of the protons when they emerge?

$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} = \frac{(1.602\times10^{-19})(1.5)}{(2\pi)(1.67\times10^{-27})} = 2.29\times10^7 \ \text{Hz}$$

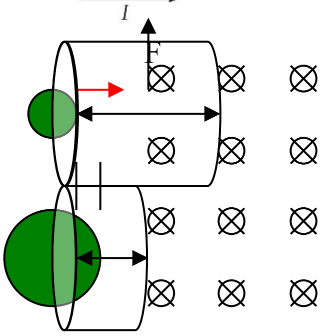
$$K = \frac{q^2B^2}{2m}r^2 = \frac{(1.602\times10^{-19})^2(1.5)^2}{2(1.67\times10^{-27})}(0.5)^2 = 4.31\times10^{-12}\,J = 26.9\,MeV$$

28.4 Magnetic Force Acting on a Current-Carrying Conductor

When a lot of electrons are moving in a confined space of a metal wire, there is a force on the wire.

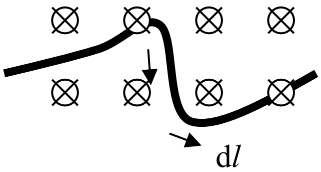


$\vec{F} = (q\vec{v} \times \vec{B})nAL$ ($\vec{J} = nq\vec{v}$ & $I = JA$) --> $\vec{F} = I\vec{L} \times \vec{B}$



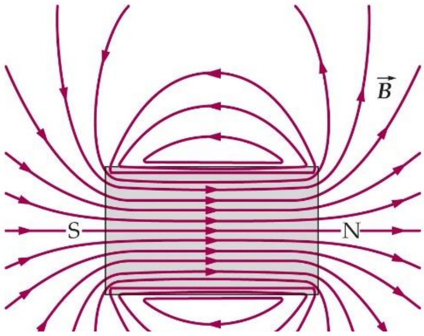
$\vec{F} = q\vec{v} \times \vec{B}$: Coulomb X Velocity X Magnetic Field -->

$\vec{F} = I\vec{L} \times \vec{B}$: (Coulomb / Time) X Length X Magnetic Field

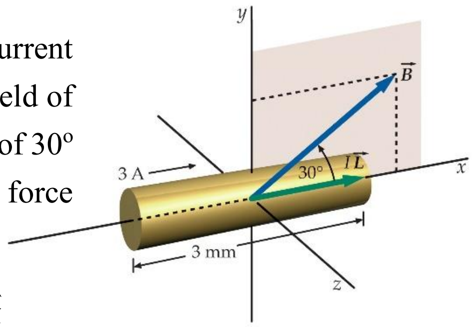


$d\vec{F} = Id\vec{l} \times \vec{B}$ ($Id\vec{l}$ is called a current element.) Use integration to obtain the total force exerted on the curved wire.

--> $\vec{F} = I\int d\vec{l} \times \vec{B}$

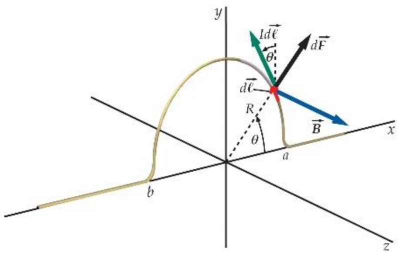


Example: A wire segment 3 mm long carries a current of 3 A in the +x direction. It lies in a magnetic field of 0.02 T that is in the xy plane and makes an angle of 30° with the +x direction. What is the magnetic force exerted on the wire segment?



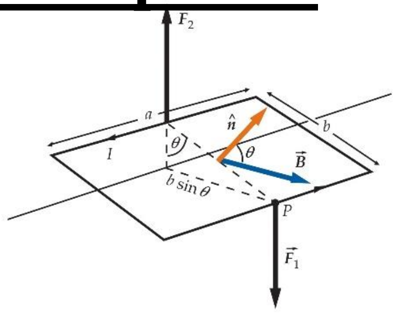
$F = I\vec{L} \times \vec{B} = 3(0.003)(0.02)\sin 30^o\,\hat{k} = 9 \times 10^{-5}\,N\hat{k}$

Example: A wire bent into a semicircular loop of radius R lies in the xy plane. It carries a current I from point a to point b. There is a uniform magnetic field $\vec{B} = B_0\hat{k}$ perpendicular to the plane of the loop. Find the force acting on the semicircular loop part of the wire. Calculate the force in the +y direction.



$$F_y = I\int RB_0 \sin\theta d\theta = IRB_0\int_0^{\pi}\sin\theta d\theta = IRB_0\int_0^{\pi} d\left(-\cos\theta\right) = 2IRB_0$$

28.5 Torque on a Current Loop in a Uniform Magnetic Field



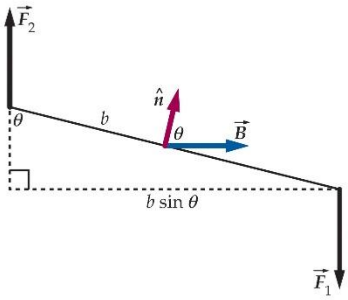
A current loop experiences no net force in a uniform magnetic field but it does experience a torque.

$$F = q\vec{v}\times\vec{B} = I\vec{L}\times\vec{B}$$

$$F_1 = F_2 = IaB$$

$$\tau = 2\left(\frac{b}{2}F\sin\theta\right) = IabB\sin\theta = \left(Iab\right)B\sin\theta$$

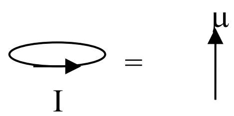
$$\vec{\tau} = \left(Iab\hat{n}\right)\times\vec{B}$$



What's the force exerted on the loop as the field is normal to the loop plane?

The current model of the magnetic dipole --> the magnetic dipole moment of the current loop:

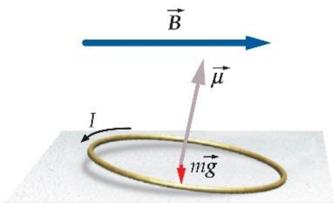
$\vec{\mu} = Iab\hat{n} = IA\hat{n}$ or $\vec{\mu} = NIA\hat{n}$ --> $\vec{\tau} = \vec{\mu}\times\vec{B}$



Compare with the electric dipole: $\vec{\tau} = \vec{p}\times\vec{E}$

What's the difference of dipole model between the electric and magnetic dipoles?

Example: A circular wire loop of radius R, mass m, and current I lies on a horizontal surface. There is a horizontal magnetic field B. How large can the current I be before one edge of the loop will lift off the surface?



$\mu = IA = I\pi R^2$, $\tau = \mu B = I\pi R^2 B = mgR$ --> $I = \dfrac{mg}{\pi RB}$

Potential Energy of a Magnetic Dipole in a Magnetic Field

When a dipole is rotated through an angle of $d\theta$, the work done is $dW = -\tau d\theta$.

$dW = -\mu B \sin\theta d\theta$

The work done on the dipole is stored in potential energy, $dU = -dW$.

$dU = \mu B \sin\theta d\theta$

$$U = \int dU = \int \mu B \sin\theta d\theta = -\mu B \cos\theta + U_0$$

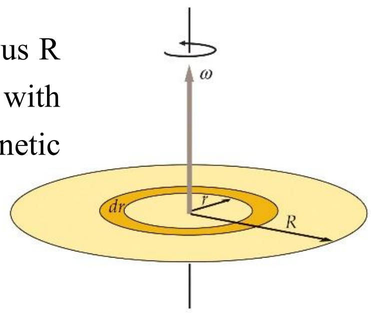
Assume $U = 0$ when $\theta = 90^o$ --> $U_0 = 0$ & $U = -\vec{\mu} \cdot \vec{B}$

Example: A square 12-turn coil with edge-length 40 cm carries a current of 3 A. It lies in the xy plane in a uniform magnetic field $\vec{B} = 0.3T\hat{i} + 0.4T\hat{k}$. Find (a) the magnetic moment of the coil, (b) torque, and (c) the potential energy.

$$\vec{\mu} = 12(3)(0.4)^2\hat{k} = 5.76\hat{k} \quad \text{Am}^2$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \quad U = -\vec{\mu} \cdot \vec{B}$$

Example: A thin nonconducting disk of mass m and radius R has a uniform surface charge per unit area σ and rotates with angular velocity $\vec{\omega}$ about the axis. Find the magnetic moment.

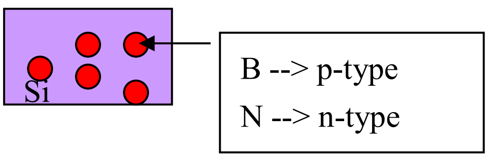


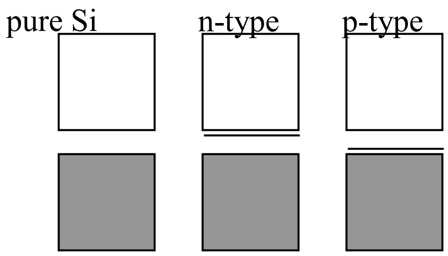
$\mu = IA$ --> $d\mu = d(IA) = (dI)A$

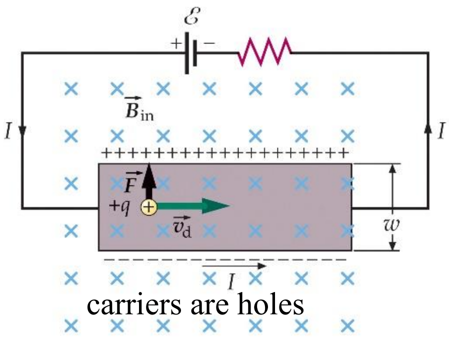
$$d\mu = \left(\frac{\sigma 2\pi r dr}{T}\right)(\pi r^2) = \left(\frac{\omega}{2\pi}\right)(\sigma 2\pi r dr)(\pi r^2) = \pi\omega\sigma r^3 dr$$

$$\mu = \int_0^R \pi\omega\sigma r^3 dr = \frac{\pi\omega\sigma}{4}R^4$$

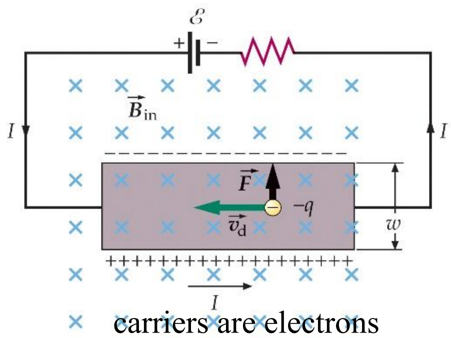
28.6 The Hall Effect







carriers are holes



carriers are electrons

What do you mean a hole?

$F = qv_d B = qE_H$ --> $E_H = v_d B$

The Hall voltage is $V_H = E_H w = v_d Bw$.

$I = AJ = A(nqv_d)$ -> $V_H = Bwv_d = Bw\dfrac{I}{nqA} = \dfrac{IBw}{nqA} = \dfrac{IB}{nqt}$

$R = \dfrac{V_H}{I} = \dfrac{B}{nqt} = R_H \dfrac{B}{t}$

$R_H = 1/nq$ is the Hall coefficient.

$n = \dfrac{IB}{V_H qt}$

By measuring the Hall effect, you can obtain:

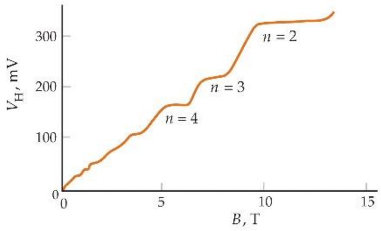
1. the carriers are positively or negatively charged
2. the carrier concentration n

Example: A silver slab of thickness 1 mm and width 1.5 cm carries a current of 2.5 A in a region in which there is a magnetic field of magnitude 1.25 T perpendicular to the slab. The Hall voltage is measured to be 0.334 μv. Calculate the number density of the charge carriers.

$$n = \frac{IB}{teV_H} = \frac{(2.5)(1.25)}{(0.001)(1.602\times10^{-19})(0.334\times10^{-6})} = 5.85\times10^{28}\,\text{electrons/m}^3$$

Hall-Effect devices are used to measure the magnetic field --> determine the current inside the wire

The Quantum Hall Effects



$$V_H = E_H w = v_d B w$$

the Hall resistance: $R = \frac{V_H}{I} = \frac{v_d B w}{(nqv_d)wt} = \frac{B}{nqt} == \frac{1}{n}\left(\frac{h}{e^2}\right)$

the Hall coefficient: $R_H = \frac{E_H}{JB} = \frac{v_d B}{(nqv_d)B} = \frac{1}{nq}$