Chapter 29 Sources of the Magnetic Field

The earliest known sources of magnetism were permanent magnets. Oersted discovered that a compass needle was deflected by an electric current --> current model of magnetism

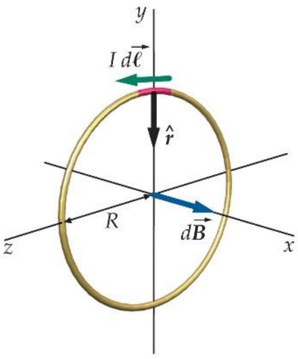
29.1 The Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$$

$$\vec{B} = \int \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$$

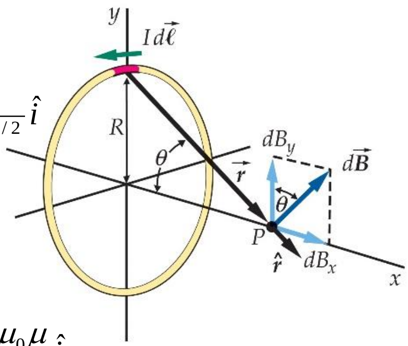
The sources of magnetic fields are electric currents and moving charges.

$\vec{B}$ Due to a Current Loop



The magnetic field due to the entire loop at the center of the loop is:

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{R d\theta}{R^2}\hat{i} = \frac{\mu_0 I}{2R}\hat{i}$$



The magnetic field at a point on the axis:

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{R d\theta}{\left(x^2 + R^2\right)} \frac{R}{\sqrt{x^2 + R^2}}\hat{i} = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}\hat{i}$$

At great distances from the loop (magnetic dipole):

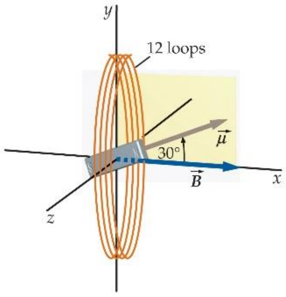
$x >> R$ --> $\vec{B} = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}\hat{i} \sim \frac{\mu_0 I R^2}{2x^3}\hat{i} = \frac{\mu_0 I \pi R^2}{2\pi x^3}\hat{i} = \frac{2\mu_0 \mu}{4\pi x^3}\hat{i}$

Compare with the electric dipole: $E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{x^3}$ (monopole: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$)

Example: A circular coil of radius 5.0 cm has 12 turns and lies in the x = 0 plane and is centered at the origin. It carries a current of 4.0 A so that the direction of the coil is along the x-axis. Find the magnetic field on the axis at (a) x = 0, (b) x = 15 cm, and (c) x = 3 m.

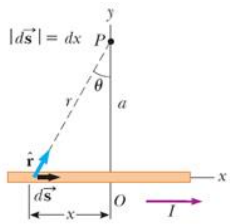
$$\vec{B} = \frac{\mu_0 I}{2R}\hat{i} = \frac{(4\pi\times10^{-7})(12)(4.0)}{2(0.05)} = 6.03\times10^{-4}$$

$$\vec{B} = \frac{\mu_0 IR}{2(x^2+R^2)}\frac{R}{\sqrt{x^2+R^2}}\hat{i}$$



Example: Find the torque on the magnet.

$$\vec{B} = \frac{\mu_0 I}{2R}\hat{i}\ ,\quad \vec{\tau} = \vec{\mu}\times\vec{B}$$

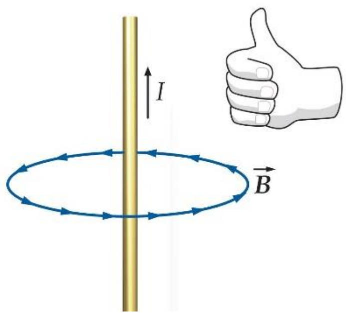


$\vec{B}$ Due to a Current in a Straight Line

$$\int dB = \frac{\mu_0 I}{4\pi}\int\frac{dx}{(a^2+x^2)}\left|d\hat{l}\times\hat{r}\right| = \frac{\mu_0 I}{4\pi}\int\frac{dx}{(a^2+x^2)}\frac{a}{\sqrt{a^2+x^2}}$$

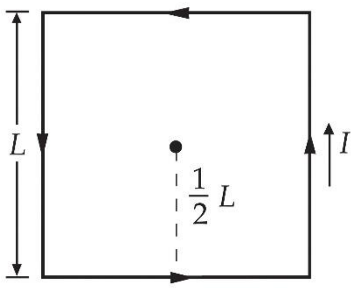
let $x = a\tan\theta$

$$B = \frac{\mu_0 I}{4\pi}\int\frac{a^2\sec^2\theta d\theta}{a^3\sec^3\theta} = \frac{\mu_0 I}{4\pi a}(\sin\theta_2 - \sin\theta_1)$$



For a infinite wire, $\theta_1 = -\frac{\pi}{2}$ & $\theta_2 = \frac{\pi}{2}$

--> $B = \frac{\mu_0 I}{2\pi R}$



Example: Find the magnetic field at the center of a square loop of edge length L = 50 cm, which carries a current of 1.5 A.

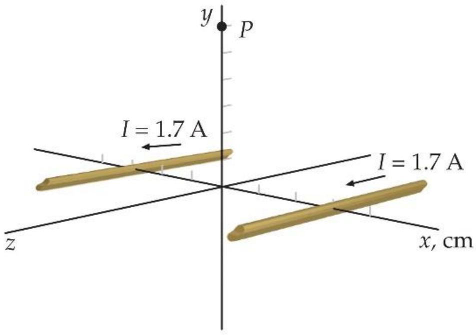
For one edge, $\theta_1 = -\frac{\pi}{4}$ & $\theta_2 = \frac{\pi}{4}$

$$B = \frac{\mu_0 I}{4\pi R}(\sin\theta_2 - \sin\theta_1) = \frac{\mu_0 I}{4\pi R}\sqrt{2}$$

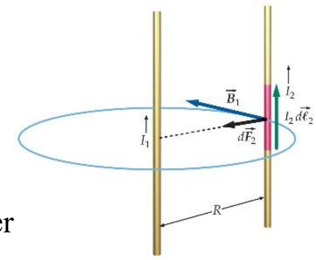
Total magnetic field: $B = 4\frac{\mu_0 I}{4\pi R}\sqrt{2}$

Example: The current is 1.7 A. Find the magnetic field at y = 6 cm on the y-axis.

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$



29.2 The Magnetic Force Between Two Parallel Conductors



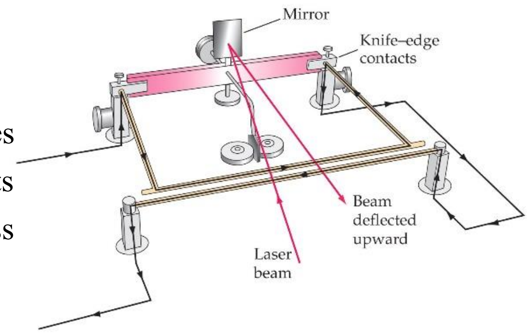
Lorentz force is more fundamental concept.
It's not good idea to take them as two magnets or to consider from the view point of magnetic field.

1. Evaluate the magnetic field due to one of the two conductors.
2. Apply the Lorentz force law to obtain the magnetic force on the other conductor.

$$(\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}) \quad \& \quad (\vec{F} = I\vec{L}\times\vec{B} \;\text{-->}\; d\vec{F} = Id\vec{l}\times\vec{B})$$

$dF_2 = I_2 dl_2 \dfrac{\mu_0 I_1}{2\pi R}$ --> cannot differentiate I_1 from I_2 --> mutual inductance

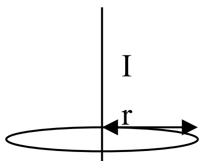
Current or Ampere meter:



Example: Two straight rods 50-cm long with axes 1.5 mm apart in a current balance carry currents of 15 A each in opposite direction. What mass must be placed on the upper rod?

$$F = ILB = IL\frac{\mu_0 I}{2\pi R} = mg$$

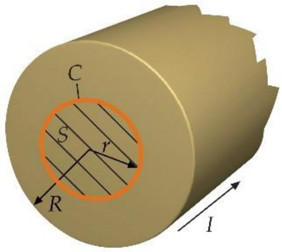
29.3 Ampere's Law



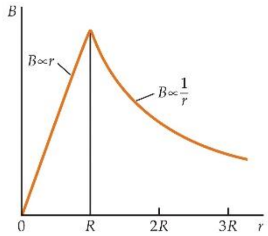
The current is the source of a magnetic field.

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}, \quad \oint \vec{B}\cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi r}\hat{\phi}\cdot\left(rd\phi\hat{\phi}\right) = \int_0^{2\pi}\frac{\mu_0 I}{2\pi r}rd\phi = \mu_0 I$$

Ampere's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ --> $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$



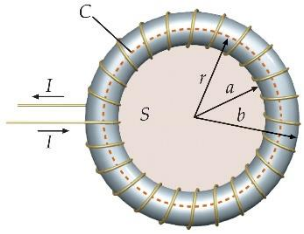
Example: A long, straight wire of radius R carries a current I that is uniformly distributed over the circular cross section of the wire. Find the magnetic field both outside and inside the wire.



$I = \pi R^2 J$ --> $J = \dfrac{I}{\pi R^2}$

$r > R$ --> $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I$ --> $\vec{B} = \dfrac{\mu_0 I}{2\pi r}\hat{\phi}$

$r < R$ --> $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I_{enc} = \mu_0 J\pi r^2 = \mu_0 I\dfrac{r^2}{R^2}$ --> $\vec{B} = \dfrac{\mu_0 I r}{2\pi R^2}\hat{\phi}$



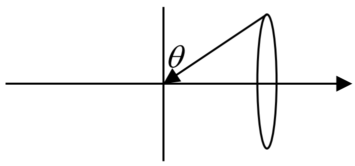
Toroid:

$r < a$ --> $\oint \vec{B}\cdot d\vec{l} = B2\pi r = 0$ --> $\vec{B} = 0$

$r > b$ --> $\oint \vec{B}\cdot d\vec{l} = B2\pi r = 0$ --> $\vec{B} = 0$

$b > r > a$ --> $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0(NI)$ --> $\vec{B} = \dfrac{\mu_0 NI}{2\pi r}\hat{\phi}$

29.4 The Magnetic Field of a Solenoid

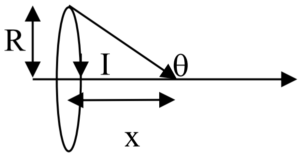


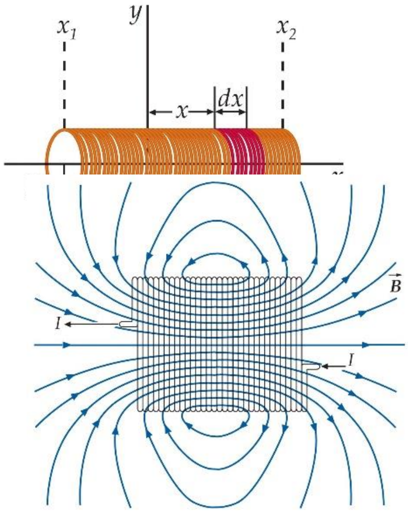
$$\vec{B} = \hat{i}\,\frac{\mu_0 n I R^2}{2}\int_{x1}^{x2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} \quad \text{let} \quad x = R\tan\theta$$

$$\vec{B} = \hat{i}\,\frac{\mu_0 n I R^2}{2}\int_{\theta 1}^{\theta 2}\frac{\cos\theta d\theta}{R^2} = \hat{i}\,\frac{\mu_0 n I}{2}\left(\sin\theta_2 - \sin\theta_1\right)$$

For a infinite lone solenoid: $\theta_1 = -\pi/2$ & $\theta_2 = \pi/2$ --> $\vec{B} = \hat{i}\,\mu_0 n I$

The magnetic field lines of a solenoid are identical to that of a bar magnet.





$$\vec{B} = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}\hat{i}$$

$$d\vec{B} = \frac{\mu_0 R^2 I n dx}{2\left(x^2 + R^2\right)^{3/2}}\hat{i}$$

--> $\vec{B} = \hat{i}\int_{x1}^{x2} \frac{\mu_0 R^2 I n dx}{2\left(x^2 + R^2\right)^{3/2}} = \hat{i}\,\frac{\mu_0 n I R^2}{2}\int_{x1}^{x2} \frac{dx}{\left(x^2 + R^2\right)^{3/2}}$

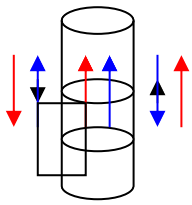
let $x = R\cot\theta$

$$\vec{B} = \hat{i}\,\frac{\mu_0 n I R^2}{2}\int_{\theta 1}^{\theta 2} \frac{-\sin\theta d\theta}{R^2} = \hat{i}\,\frac{\mu_0 n I}{2}\left(\cos\theta_2 - \cos\theta_1\right)$$

For a infinite lone solenoid: $\theta_1 = \pi$ & $\theta_2 = 0$ --> $\vec{B} = \hat{i}\,\mu_0 n I$

Apply Ampere's Law

Example: Find the magnetic field of a very long solenoid, considering of n closely wound turns per unit length on a cylinder of radius R and carrying a steady current I.



We know that B does not dependent on the distance from the surface for the surface current.

$BL = \mu_0 n I L$ → $\vec{B}_{inside} = \mu_0 n I \hat{z}$

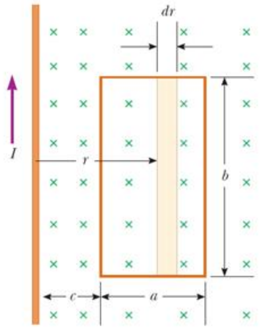
Limitations of Ampere's Law

Ampere's law is useful for calculating the magnetic field only when the current is steady and the geometry has a high degree of symmetry.

29.5 Gauss's Law for Magnetism

Gauss's Law Applied to Calculation of Magnetic Flux on an Open Area:

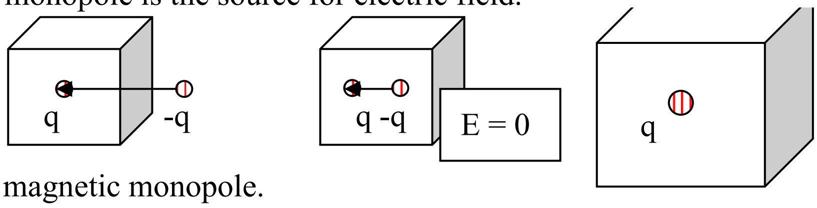
$\Phi_B = \int \vec{B}\cdot d\vec{A}$ or $\Phi_B = BA\cos\theta$



Example: Magnetic Flux Through a Rectangular Loop

$$\Phi_B = \int_c^{c+a}\int_0^b \frac{\mu_0 I}{2\pi r} dz dr = \frac{\mu_0 I b}{2\pi}\ln\left(\frac{c+a}{c}\right)$$

Charge or electric monopole is the source for electric field.



You cannot find a magnetic monopole.

$$\phi_{m,net} = \oint \vec{B}\cdot d\vec{a} \propto number\_of\_enclosed\_monopoles = Nq_m + N(-q_m) = 0$$

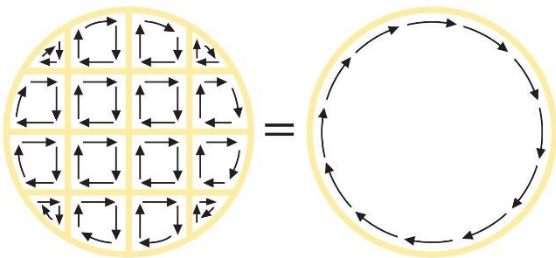
29.6 Magnetism in Matter

Magnetization and Magnetic Susceptibility

magnetization $\vec{M}$: the net magnetic dipole moment ($\vec{\mu}_{net}$) per unit volume of the material

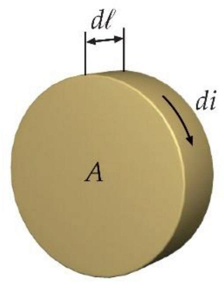
$$\vec{M} = \frac{\vec{m}}{V} = \frac{\vec{\mu}_{net}}{V} = \frac{d\vec{\mu}_{net}}{dV}$$

Current Loop Picture:

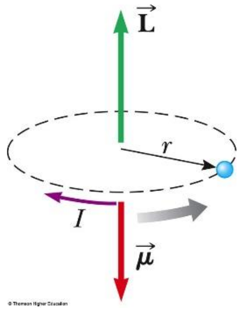


Will you be hurt by the large current flowing in a permanent bar magnet?

$d\mu = AdI$, $M = \frac{d\mu}{dV} = \frac{AdI}{Adl} = \frac{dI}{dl}$



The applied magnetic field is used to align small magnets in materials so the total magnetic field is higher than the applied filed.



Atomic Magnetic Moments

Angular momentum of orbital electrons in an atom:

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times (m\vec{v}) = mrv$$

Is there any relation to magnetic dipole moment?

$$\vec{\mu} = IA = I\pi r^2 = \frac{q}{2\pi r / v} \pi r^2 = \frac{1}{2} qrv \text{ --> } \mu = \frac{q}{2m} L$$

Magnetism, magnetic moment, and magnetization are in consequences of orbital motion of charged particles.

The angular momentum and magnetic moment are in the same direction:

$$\vec{\mu} = \frac{q}{2m} \vec{L}$$

Bohr's model: The angular momentum is quantized with a unit of $\hbar$, so we express the magnetic moment in terms of $n = \frac{\vec{L}}{\hbar}$ (a number). $\vec{\mu} = \frac{q\hbar}{2m} \frac{\vec{L}}{\hbar} = n \frac{q\hbar}{2m} = n\mu_B$

For electrons: $\vec{\mu} = \frac{q\hbar}{2m} \frac{\vec{L}}{\hbar} = \frac{-e\hbar}{2m_e} \frac{\vec{L}}{\hbar} = -n\mu_B$

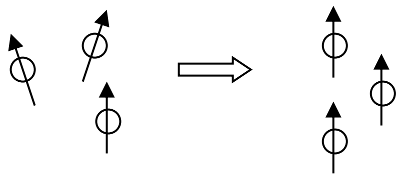
The unit of magnetization in atomic scale is Bohr magneton $\mu_B = \frac{e\hbar}{2m_e}$.

The intrinsic spin angular momentum $\vec{S}$ generates the magnetic moment:

$$\vec{\mu}_s = -2\mu_B \frac{\vec{S}}{\hbar}$$

If all the atoms have the same magnetic moment aligned in the same direction. The **saturated magnetization** is:

$$\vec{m} = \vec{\mu} = N\vec{\mu}_s \text{ or } \vec{M}_s = \frac{\vec{m}}{V} = \frac{N}{V} \vec{\mu}_s = \vec{\mu}_s$$



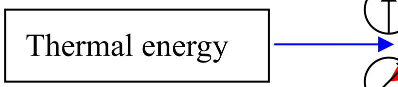
When will the magnetization be saturated?

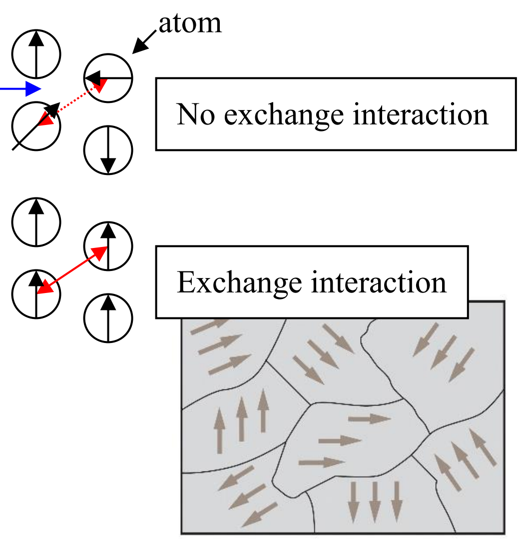
Paramagnetism

1. The potential energy of the magnetic moment in a magnetic field is $U = -\vec{\mu} \cdot \vec{B}$.
2. The magnetic moment of electrons in atoms is quantized to be $l\mu_B$ and the potential energy is quantized to be $m_l \mu_B B$, where n can be positive and negative integer.
3. When you turn on the magnetic field, the magnetic moment of atoms could have different potential energy.

Ferromagnetism

What is the exchange interaction?





What is a magnetic domain?

What is the Curie temperature?

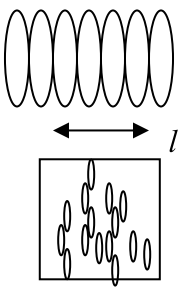
A long iron rod placing in a solenoid:

$$B = B_{app} + \mu_0 M = \mu_0 nI + \mu_0 M$$

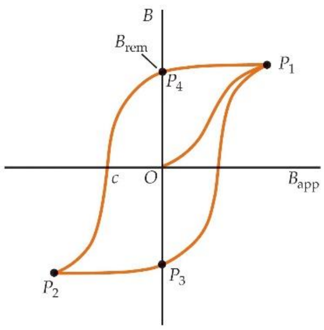
$$(M = \frac{N}{V}\mu = \frac{N}{V} IA = \frac{N}{l} I = n' I)$$

A solenoid:

A magnet:



What is a hysteresis loop?



Diamagnetism

What's the diamagnetism? --> Lorentz law
What's the Lenz law?

The magnetic field in the **superconductor**:

$B = (1+\chi_m)B_{app} = 0$ --> $\chi_m = -1$

29.7 The Magnetic Field of the Earth