Chapter 30 Faraday's Law

Change moving --> current --> magnetic field (static current --> static magnetic field)

The source of magnetic fields is current.
The source of electric fields is charge (electric monopole).

Alternating magnetic field --> change the flux --> emf
Changing magnetic flux --> induced **emf** --> induced currents
Motion of a conductor in a magnetic field --> motional **emf**

electric flux: EA --> $\Phi_E = \int \vec{E} \cdot d\vec{A}$ <--> Gauss's Law: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

magnetic flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$ <--> Gauss's Law: $\oint \vec{B} \cdot d\vec{A} = 0$

Unit of magnetic flux: weber (T m^2)

30.1 Faraday's Law of Induction

Experimentalists, Faraday and Henry et al., found that a emf is induced in the circuit if the magnetic flux through the surface bounded by the circuit is changed.

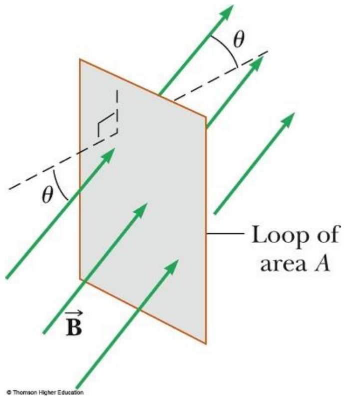
The ways for changing the magnetic flux:
1. the currents that generate the magnetic field increase or decrease
2. the permanent magnet moves forward or backward
3. the circuit is rotating in a static magnetic field
4. the circuit is moving in a nonuniform magnetic field

5. the area of the circuit is increasing or decreasing

Faraday's law:

$$\varepsilon = -\frac{d\Phi_m}{dt}$$

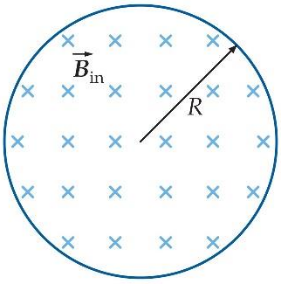
$$\varepsilon = -\frac{d}{dt}\left(\vec{B}\cdot\vec{A}\right) = -\frac{d}{dt}\left(BA\cos\theta\right)$$



Example: A uniform magnetic field makes an angle of 30º with the axis of a circular coil of 300 turns and a radius of 4 cm. The magnitude of the magnetic field increases at a rate of 85 T / s while its direction remains fixed. Find the magnitude of the induced emf in the coil.

$$\Phi_m = \vec{B}\cdot\vec{A} = NB\pi r^2\cos\theta\,,\quad \varepsilon = -\frac{d\Phi_m}{dt} = -N\left(\frac{dB}{dt}\right)\pi r^2\cos\theta$$

Example: A magnetic field $\vec{B}$ is perpendicular to the plane of the page. $\vec{B}$ is uniform throughout a circular region of radius R. Outside this region, B equals zero. The direction of $\vec{B}$ remains fixed and the rate of change of B is dB/dt. What are the magnitude and direction of the induced electric field in the plane of the page (a) a distance r < R from the center of the region and a distance r > R from the center, where B = 0.

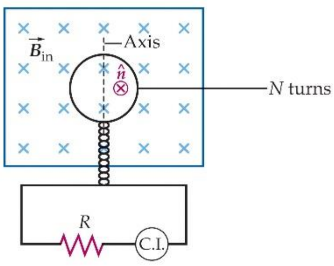


$$r < R:\ \ \Phi_m = B\pi r^2\,,\ \ \oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}\ \ \text{-->}\ \ 2\pi r E = \pi r^2\left(-\frac{dB}{dt}\right)$$

$$r > R:\ \ \Phi_m = B\pi R^2\,,\ \ \oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}\ \ \text{-->}\ \ 2\pi r E = \pi R^2\left(-\frac{dB}{dt}\right)$$

Compare with the Ampere's law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}\,,\ \ 2\pi r B = \mu_0 I_{enc}$

Example: A small coil of N turns has its plane perpendicular to a uniform static magnetic field $\vec{B}$. The coil is connected to a current integrator. Find the charge passing through the coil if the coil is rotated through 180º about the axis.

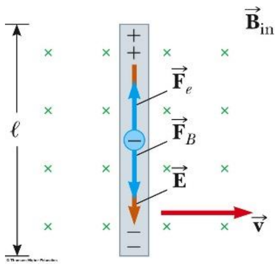


$$\Phi_m = NBA\cos\theta\,,\quad IR = R\frac{dQ}{dt} = \varepsilon = -\frac{d\Phi_m}{dt} = NBA\sin\theta\frac{d\theta}{dt}$$

$$dQ = \frac{NBA\sin\theta}{R}d\theta \quad \text{-->} \quad Q = \frac{NBA}{R}\int_0^{\pi}\sin\theta d\theta = \frac{2NBA}{R}$$

30.2 Motional EMF

The emf induced in a conductor moving through a magnetic field is called motional emf.

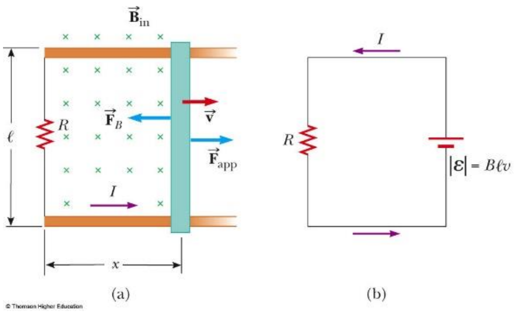


A conducting rod moves in a magnetic field.

$$qE = qvB$$

$$E = vB$$

$$\Delta V = El = vBl$$



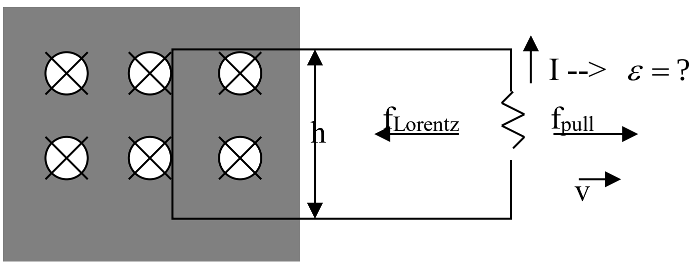
$$\Phi_B = Blx$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -Bl\frac{dx}{dt}$$

$$\varepsilon = -Blv$$

$$I = \frac{|\varepsilon|}{R} = \frac{Blv}{R}$$

Phenomena of The Induced emf and Currents:



1. You use the force f_{pull} to pull the wire.
2. You found that the wire is moving at a constant velocity v.
3. There must be one opposite force that cancels the pulling force.
4. The force generated by the magnetic field and doing on the (moving) charge may be Lorentz force $f_{Lorentz}$.

5. The electric field that induced the current I or drive the charge q move must be related to the Lorentz force.
6. The charge is moving to the right at a constant speed of v. The charge will be exerted with a Lorentz force of $f = qvB = qE$ that drive the current I to flow.
7. The driving electric potential of the induced current I is $V = El = vBl$

Example: A rod of mass *m* and resistance *R* slides on frictionless conducting rails with a separation distance of *l* in a region of static uniform magnetic field *B*. An external agent is pushing the rod, maintaining its motion to the right at constant speed v_0. At time t = 0, the agent abruptly stops pushing the rod continuous forward. The rod is slowed down by the magnetic force. Find the speed v of the rod as a function of time.

$F = -qvB = -IBl$, $I = \frac{\varepsilon}{R} = \frac{vBl}{R}$ --> $F = ma = -IBl = -\frac{B^2l^2}{R}v$

--> Differential Eq: $m\frac{dv}{dt} = -\frac{B^2l^2}{R}v$

Solve the differential eq: $v = v_0 e^{-\frac{B^2l^2}{mR}t}$

The general equation for motional emf:

$V = vBl = (\vec{v} \times \vec{B}) \cdot \vec{l}$ --> $dV = (\vec{v} \times \vec{B}) \cdot d\vec{l}$ --> $\oint dV = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$

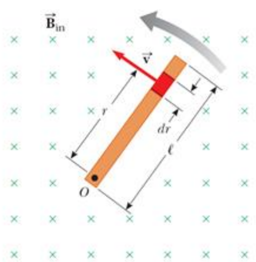
In static magnetic field & without rotation:

$$\oint dV = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = 0$$

Otherwise:

$$\varepsilon = \oint dV = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = -\frac{d\Phi_m}{dt}$$

Example: Motional emf induced in a rotating bar.



$$dV = (\vec{v} \times \vec{B}) \cdot d\vec{l} = (-\omega r B)dr$$

$$\varepsilon = -\int_0^l \omega r B dr - \frac{1}{2}\omega B l^2$$

30.3 Lenz’s Law

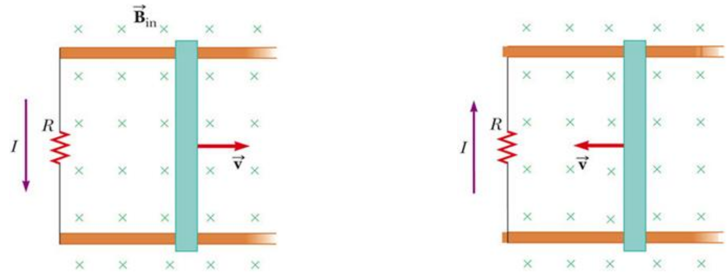
Lenz’s Law:

The induced emf is in such a direction as to opposite, or tend to oppose, the change that produces it.

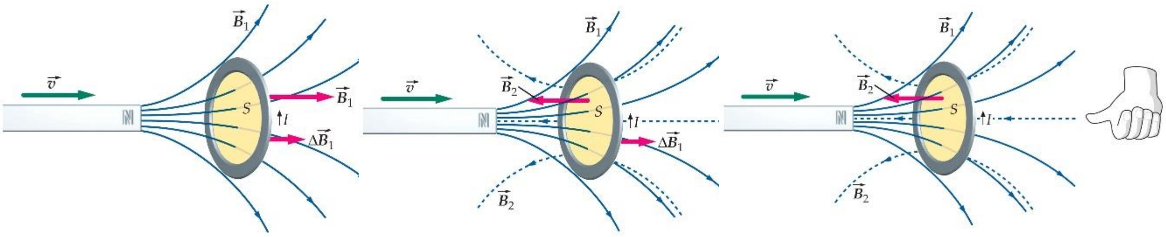
The word “**change**” is a key word in the expression of the Lenz’s law.

When a magnetic flux through a surface changes, the magnetic field due to any induced current produces a flux of its own – through the same surface and in opposite to the change.

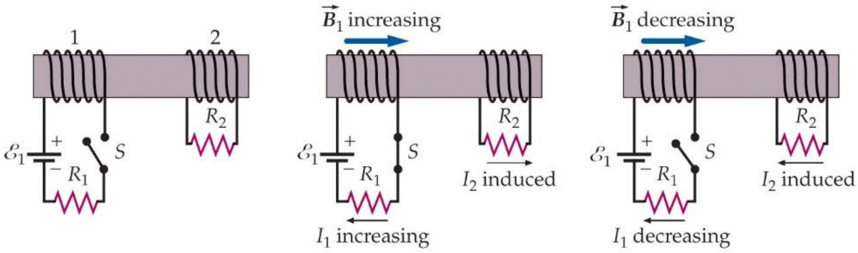
Does the induced emf still exist when no circular loop is placed?



Example: Find the direction of the induced current in the loop.

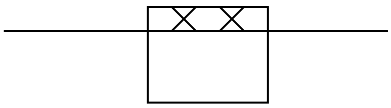


The magnetic flux changes by increasing or decreasing the current:



Example: A rectangular coil of N turns, each of width a and length b; where N = 80, a = 20 cm, and b = 30 cm; is located in a magnetic field B = 0.8 T directed into the page, with only half of the coil in the region of the magnetic field. The resistance R of the coil is 30 Ω. Find the magnitude and direction of the induced current if the coil is moved with a speed of 2 m/s (a) to the right, (b) up, and (c) down.

(a) 0

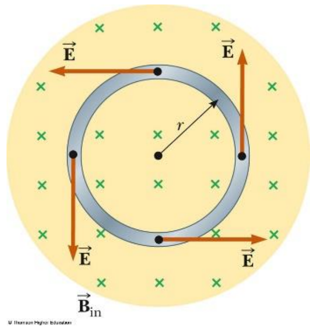


(b) $\Phi_m = NBA = NBax$, $\varepsilon = -\dfrac{d\Phi_m}{dt} = -NBa\dfrac{dx}{dt}$

up: $\dfrac{dx}{dt} > 0$, down: $\dfrac{dx}{dt} < 0$ --> $I = \dfrac{\varepsilon}{R}$

30.4 Induced emf and Electric Fields

Work: $q\varepsilon = Fs = qE2\pi r$ → $\varepsilon = E2\pi r$



$$\Phi_B = BA = B\pi r^2, \quad \varepsilon = -\frac{d\Phi_B}{dt} = -\pi r^2\frac{dB}{dt}$$

$$E = \frac{\varepsilon}{2\pi r} = -\frac{\pi r^2}{2\pi r}\frac{dB}{dt} = -\frac{r}{2}\frac{dB}{dt}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{r}{2}\frac{dB}{dt}2\pi r = -\frac{d\Phi_B}{dt}$$

Example: A long solenoid of radius *R* has *n* turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{\max}\cos(\omega t)$, where $I_{\max}$ is the maximum current and ω is the angular frequency of the ac current source. (a) Determine the magnitude of the induced electric field outside the solenoid at a distance $r > R$ from its long central axis. (b) What is the magnitude of the induced electric field inside the solenoid, a distance *r* from its axis?

$$B_{in} = \mu_0 n I_{\max}\cos(\omega t)$$

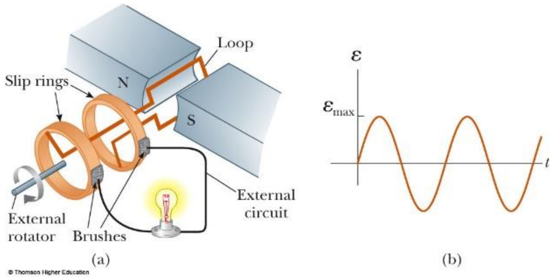
(a)

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B \rightarrow 2\pi r E = -\frac{d}{dt}\left(\pi R^2\mu_0 n I_{\max}\cos(\omega t)\right)$$

(b)

$$2\pi r E = -\frac{d}{dt}\left(\pi r^2\mu_0 n I_{\max}\cos(\omega t)\right)$$

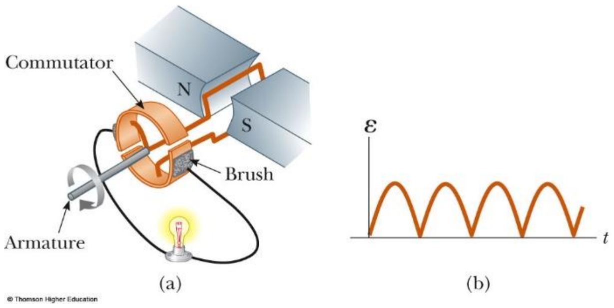
30.5 Generators and Motors



$$\Phi_B = BA\cos\theta$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = NBA\sin\theta\frac{d\theta}{dt} = NBA\omega\sin\theta$$

DC generator:



30.6 Eddy Currents

A changing flux sets up circulating currents --> eddy currents (Lenz's law)

1. in the transformer, eddy current which generates Joule heating should be prevented
2. eddy current in induction cooker (induction oven)
3. eddy current is used for rapid heating

What is induction heating?

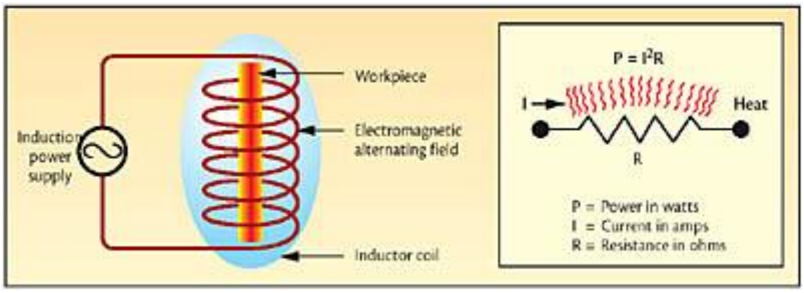


Fig 1 Induction heating is a noncontact heating method
Fig 2 Heat energy (E) produced in an electric circuit is equal to I2 _ R.

Induction heating (Fig. 1) is a noncontact heating method; one in which an electrically conductive material (typically a metal) is heated by an alternating magnetic field. Invisible lines of force are created by a work coil when a current flows through it, the result of which is an induced current in the conductive workpiece. Heating results due to the Joule effect and, to a lesser degree, magnetic hysteresis (i.e., power loss other than by eddy currents in a magnetic material caused by reversals of the magnetic field). Joule's Law (Fig. 2) states that the rate at which heat energy is produced in any part of an electric circuit is measured by the product of the square of the current (I) times the resistance (R) of that part of the circuit.

Ref: http://www.industrialheating.com/CDA/ArticleInformation/features/BNP__Features__Item/0,2832,124816,00.html

4. eddy current is used to damp mechanical vibration