Chapter 32 Alternating Current Circuits

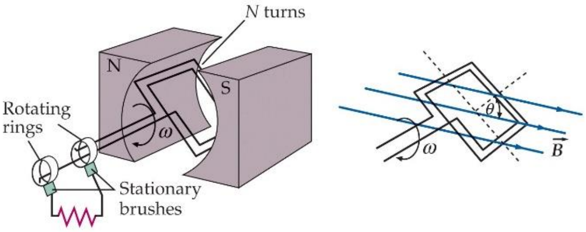
Most of the electrical energy is produced by electrical generators in the form of sinusoidal alternating current.

Why do we use the sinusoidal electric potential but neither the triangular nor the square waves?

32.1 AC Sources

$\Phi_m = BA$, $\varepsilon = -\frac{d\Phi_m}{dt} = -\frac{dB}{dt}A\cos\theta + B\frac{dA}{dt}\cos\theta + BA\frac{d\cos\theta}{dt}$

What's the best and the cheapest way to make the flux change?



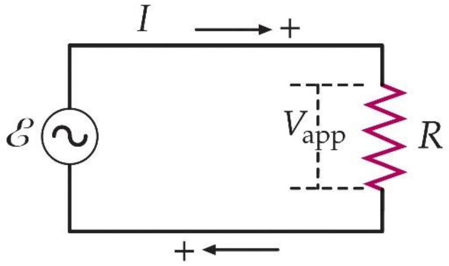
$\Phi_m = NBA\cos\theta$

if the angular velocity is ω

--> $\Phi_m = NBA\cos(\omega t + \delta)$

$$\varepsilon = -\frac{d\Phi_m}{dt} = \omega NBA\sin(\omega t + \delta) = \varepsilon_{peak}\sin(\omega t + \delta)$$

32.2 Resistors in an AC Circuit



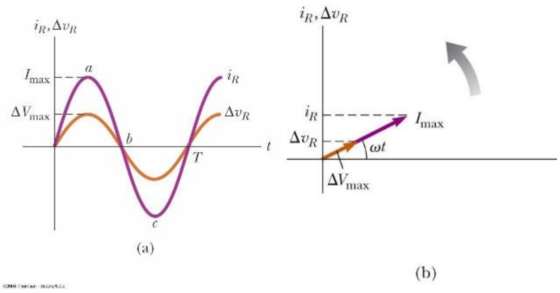
$V_R = \varepsilon = \varepsilon_{peak}\sin(\omega t + \delta)$

Let $\delta = \frac{\pi}{2}$ --> $\sin(\omega t + \delta) = \cos(\omega t)$

$V_R = V_{R,peak}\cos(\omega t)$

Applying Ohm's law (What is the Ohm's law? Why do we use this? --> Linear relation between I and V)

$$V_R = V_{R,peak}\cos(\omega t) = IR \quad \text{-->} \quad I = \frac{V_{R,peak}}{R}\cos(\omega t) = I_{peak}\cos(\omega t)$$



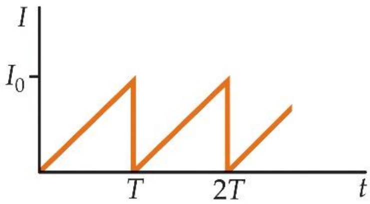
Root Mean Square Values

$$I_{rms} = \sqrt{\left(I_{\text{instantaneous}}{}^2\right)_{av}}\ , \quad I_{\text{instantaneous}} = I_{peak}\cos(\omega t)$$

$$\left(I_{\text{instantaneous}}{}^2\right)_{av} = \left(I_{peak}^2\cos^2(\omega t)\right)_{av} = \frac{1}{2}I_{peak}^2$$

$$I_{rms} = \frac{1}{\sqrt{2}}I_{peak} \approx 0.707 I_{peak}$$

Example: Find the average current and the rms current for the sawtooth waveform. In the region 0<t<T, the current is given by $I = I_0 \dfrac{t}{T}$.



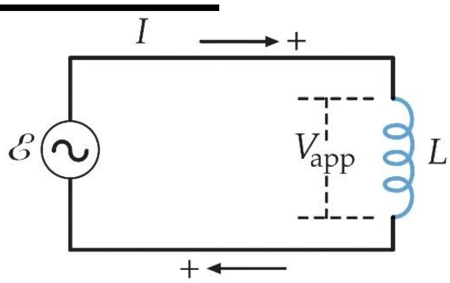
$$I_{av} = \frac{1}{T}\int_0^T I_0 \frac{t}{T}dt = \frac{I_0}{2}$$

$$\left(I^2\right)_{av} = \frac{1}{T}\int_0^T I_0^2 \frac{t^2}{T^2}dt = \frac{I_0^2}{3} \quad \text{-->} \quad I_{rms} = \frac{I_0}{\sqrt{3}}$$

32.3 Inductors in an AC Circuit

$$V_L = -L\frac{dI}{dt}$$

Apply Kirchhoff's loop rule:



$\varepsilon - L\frac{dI}{dt} = 0$ --> $\varepsilon_{peak}\cos(\omega t) - L\frac{dI}{dt} = 0$

$\varepsilon_{peak}\cos(\omega t) = L\frac{dI}{dt}$ --> $\frac{\varepsilon_{peak}\cos(\omega t)}{L}dt = dI$

--> $I = \frac{\varepsilon_{peak}}{\omega L}\sin(\omega t) = I_{peak}\sin(\omega t) = I_{peak}\cos\left(\omega t - \frac{\pi}{2}\right)$

--> $I_{peak} = \frac{\varepsilon_{peak}}{\omega L} = \frac{V_{L,peak}}{\omega L} = \frac{V_{L,peak}}{X_L}$ --> $V_{L,peak} = I_{peak}X_L$ compare with $V = IR$

$X_L = \omega L$ --> resistance like --> impedance --> inductive reactance

The resistance R is not changed with the frequency ω but the impedance may change with the frequrncy.

$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$, $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$ --> $I_{rms} = \frac{V_{rms}}{X_L}$

Instantaneous Power Delivered:

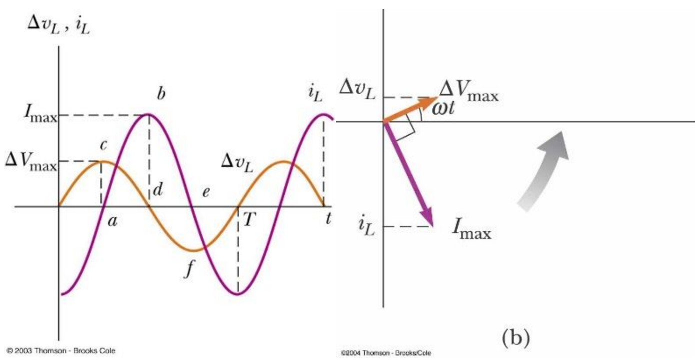
$$P = VI = (V_{peak}\cos(\omega t))(I_{peak}\sin(\omega t)) = \frac{1}{2}V_{peak}I_{peak}\sin(2\omega t)$$

The average of dissipated energy is: $\frac{1}{T}\int_0^T \frac{1}{2}V_{peak}I_{peak}\sin(2\omega t)dt = 0$. --> No energy is dissipated in an inductor.

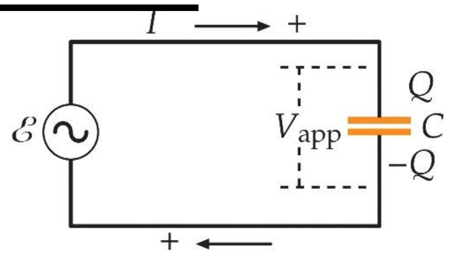
Example: Inductive Reactance

The potential drop across a 40-mH inductor is sinusoidal with a peak potential drop of 120 V. Find the inductive reactance and the peak current when the frequency is 60 Hz.

$X = \omega L = (2\pi)(60)(40\times10^{-3}) = 15.1\Omega$, $I_{peak} = \frac{V_{peak}}{X}$



32.4 Capacitos in an AC Circuit



$V = \frac{Q}{C}$, $\varepsilon - \frac{Q}{C} = 0$

--> $\varepsilon_{peak} \cos(\omega t) = \frac{Q}{C}$

--> $I = \frac{dQ}{dt} = \frac{d}{dt}\left(\varepsilon_{peak} \cos(\omega t) C\right) = -\omega \varepsilon_{peak} C \sin(\omega t) = -I_{peak} \sin(\omega t)$

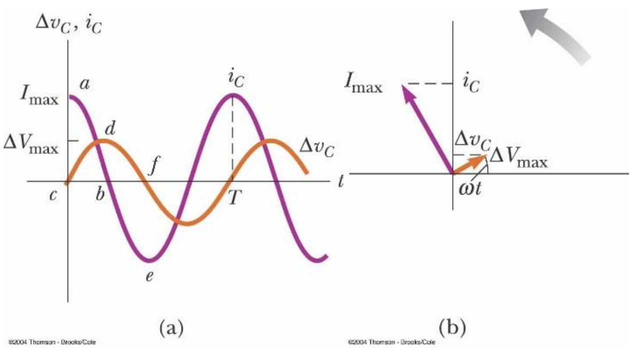
$I = -I_{peak} \sin(\omega t) = I_{peak} \cos\left(\omega t + \frac{\pi}{2}\right)$

$\varepsilon_{peak} \omega C = I_{peak}$ --> $\varepsilon_{peak} = \frac{1}{\omega C} I_{peak} = I_{peak} X$ --> $X = \frac{1}{\omega C}$

Example: Capacitive Reactance

A 20-μF capacitor is placed across an ac generator that applies a potential drop with an amplitude (peak value) of 100 V. Find the capacitive reactance and the current amplitude when the frequency is 60 Hz.

$X = \frac{1}{\omega C} = \frac{1}{2\pi(60)(20 \times 10^{-6})} = 133\Omega$, $I_{peak} = \frac{V_{peak}}{X}$



32.5 The *RLC* Series Circuit

Phasors

The current in a steady-state ac circuit varies with time as $I = I_{peak} \cos(\omega t)$

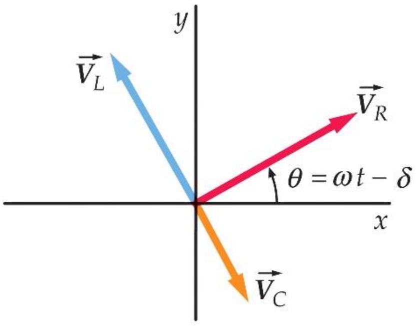
--> The voltage drop across a resistance is $V_R = IR = I_{peak} R \cos(\omega t)$.

--> The voltage drop across an inductor is $V_L = L\dfrac{dI}{dt} = -I_{peak}\omega L\sin(\omega t)$.

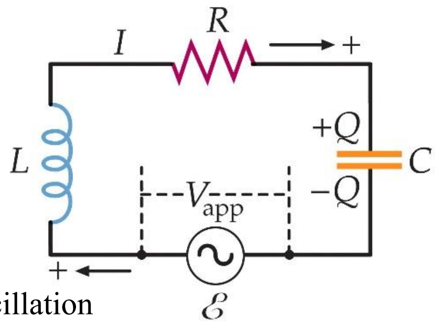
$$V_L = -I_{peak}\omega L\cos\left(\frac{\pi}{2} - \omega t\right) = I_{peak}\omega L\cos\left(\omega t + \frac{\pi}{2}\right)$$

--> The voltage drop across a capacitor is $V_C = \dfrac{1}{C}\int Idt = \dfrac{1}{\omega C}I_{peak}\sin(\omega t)$

$$V_C = \frac{I_{peak}}{\omega C}\cos\left(\frac{\pi}{2} - \omega t\right) = \frac{I_{peak}}{\omega C}\cos\left(\omega t - \frac{\pi}{2}\right)$$



Series RLC Circuit



$$V_{app} = V_{peak}\cos(\omega t)$$

Diff. Eq.: $V_{peak}\cos(\omega t) - L\dfrac{dI}{dt} - IR - \dfrac{Q}{C} = 0$ -->

$$V_{peak}\cos(\omega t) - L\frac{d^2Q}{dt^2} - R\frac{dQ}{dt} - \frac{1}{C}Q = 0$$ (forced oscillation

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$)

Forced Oscillations --> Force The Oscillator Vibrate with The Frequency ω

Let $Q = A\sin(\omega t - \delta)$

--> $V_{peak}\cos(\omega t) + L\omega^2 A\sin(\omega t - \delta) - RA\omega\cos(\omega t - \delta) - \dfrac{1}{C}A\sin(\omega t - \delta) = 0$

$$V_{peak}\cos(\omega t) + L\omega^2 A(\sin(\omega t)\cos\delta - \cos(\omega t)\sin\delta)$$

$$-RA\omega(\cos(\omega t)\cos\delta + \sin(\omega t)\sin\delta) - \frac{1}{C}A(\sin(\omega t)\cos\delta - \cos(\omega t)\sin\delta) = 0$$

$$\cos(\omega t):\ V_{peak} - L\omega^2 A\sin\delta - RA\omega\cos\delta + \frac{1}{C}A\sin\delta = 0$$

$$\sin(\omega t): \quad L\omega^2\cos\delta - R\omega\sin\delta - \frac{1}{C}\cos\delta = 0 \quad \text{-->} \quad \tan\delta = \frac{L\omega - \frac{1}{\omega C}}{R} = \frac{X_L - X_C}{R}$$

$$\text{-->} \quad A = \frac{V_{peak}\sqrt{R^2 + (X_L - X_C)^2}}{\omega\left(R^2 + (X_L - X_C)^2\right)} = \frac{V_{peak}}{\omega\sqrt{R^2 + (X_L - X_C)^2}}$$

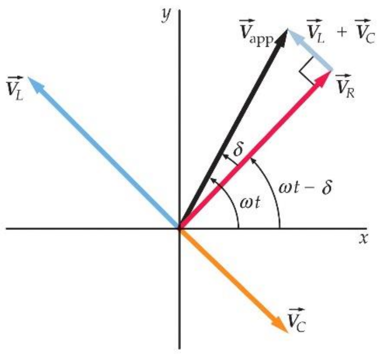
$$Q = \frac{V_{peak}}{\omega\sqrt{R^2 + (X_L - X_C)^2}}\sin(\omega t - \delta), \quad I = \frac{V_{peak}}{\sqrt{R^2 + (X_L - X_C)^2}}\cos(\omega t - \delta)$$

$$I_{peak} = \frac{V_{peak}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{peak}}{Z}$$

Total reactance: $X_L - X_C$

Impedance: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phasor:



$$V_{app} = V_{peak}\cos(\omega t) \quad \& \quad I = I_{peak}\cos(\omega t - \delta)$$

$$\text{-->} \quad V_R = I_{peak}\cos(\omega t - \delta)R$$

$$V_L = L\frac{dI}{dt} = -V_{L,peak}\sin(\omega t - \delta) = -V_{L,peak}\cos\left(\frac{\pi}{2} - (\omega t - \delta)\right) = V_{L,peak}\cos\left((\omega t - \delta) + \frac{\pi}{2}\right)$$

$$\vec{V}_{app} = \vec{V}_R + \vec{V}_L + \vec{V}_C$$

$$V_{app,peak} = \left|\vec{V}_R + \vec{V}_L + \vec{V}_C\right| = \sqrt{V_{R,peak}^2 + \left(V_{L,peak} - V_{C,peak}\right)^2}$$

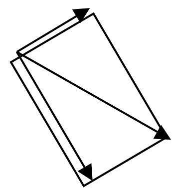
$$V_{R,peak} = I_{peak}R, \quad V_{L,peak} = I_{peak}X_L, \quad V_{C,peak} = I_{peak}X_C$$

$$\text{-->} \quad V_{app} = I_{peak}\sqrt{R^2 + (X_L - X_C)^2} = I_{peak}Z \quad \text{(Z: impedance)}$$

$$\tan\delta = \frac{\left|\vec{V}_L + \vec{V}_C\right|}{\left|\vec{V}_R\right|} = \frac{X_L - X_C}{R}$$

Example: A resistor *R* and capacitor *C* are in series with a generator. The generator applies a potential drop across the RC combination given by $V_{app} = \sqrt{2} V_{app,rms} \cos(\omega t)$.

Find the rms potential drop across the capacitor as a functional of frequency ω.



$$I = I_0 \cos(\omega t + \delta)$$

$$V_R = I_0 R \cos(\omega t + \delta)$$

$$V_C = \frac{1}{\omega C} I_0 \cos(\omega t + \delta - \pi / 2)$$

$$V_{app}^{\ 2} = I_0^{\ 2} R^2 + I_0^2 \frac{1}{(\omega C)^2} \rightarrow I_0 = \frac{V_{app}}{\sqrt{R^2 + 1/(\omega C)^2}} = \frac{\sqrt{2} V_{app,rms}}{\sqrt{R^2 + 1/(\omega C)^2}}$$

$$\rightarrow V_{out} = I_0 / \omega C = \frac{\sqrt{2} V_{app,rms}}{\sqrt{1 + (\omega C)^2 R^2}} = \sqrt{2} V_{out,rms} \rightarrow V_{out,rms} = \frac{V_{app,rms}}{\sqrt{1 + \omega^2 C^2 R^2}}$$

Parallel RLC Circuit

$$I = \sqrt{\left(\frac{\varepsilon}{R}\right)^2 + \left(\frac{\varepsilon}{\omega L} - \frac{\varepsilon}{1/\omega C}\right)^2}$$

32.6 Power in an AC Circuit

The instantaneous power dissipation:

$$P = I^2 R = I_{peak}^2 \cos^2(\omega t) R$$

The average power dissipation:

$$P_{av} = (I^2 R)_{av} = (I_{peak}^2 \cos^2(\omega t) R)_{av} = I_{peak}^2 R (\cos^2(\omega t))_{av}$$

What’s the time average? What’s the spatial average?

$$(P)_{time\_av} = \frac{1}{T} \int_0^T P dt \qquad (P)_{spatial\_av} = \frac{1}{\lambda} \int_0^\lambda P dx$$

$$\frac{1}{T}\int_0^T \cos^2\left(\frac{2\pi}{T}t\right)dt = \frac{1}{T}\int_0^T \frac{1+\cos\left(2\frac{2\pi}{T}t\right)}{2}dt = \frac{1}{T}\frac{T}{2} = \frac{1}{2}$$

$$P_{av} = I_{peak}^2 R\left(\cos^2(\omega t)\right)_{av} = \frac{1}{2}I_{peak}^2 R$$

--> $P_{av} = \frac{1}{2}I_{peak}^2 R = I_{rms}^2 R$

The average power delivered by the generator is:

$$P_{av} = (\varepsilon I)_{av} = \left(\varepsilon_{peak}\cos(\omega t) I_{peak}\cos(\omega t)\right)_{av} = \left(\varepsilon_{peak} I_{peak}\cos^2(\omega t)\right)_{av} = \frac{1}{2}\varepsilon_{peak} I_{peak}$$

$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$, $\varepsilon_{rms} = \frac{\varepsilon_{peak}}{\sqrt{2}}$ --> $P_{av} = \varepsilon_{rms} I_{rms}$

For the RLC circuit:

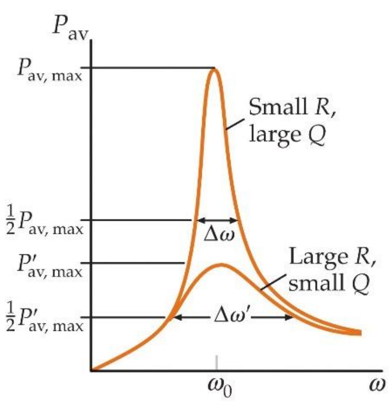
$$P = I_{peak}V_{peak}\cos(\omega t)\cos(\omega t - \phi) = I_{peak}V_{peak}\left(\cos^2(\omega t)\cos\phi + \cos(\omega t)\sin(\omega t)\sin\phi\right)$$

$$P_{avg} = I_{rms}V_{rms}\cos\phi = I_{rms}V_{rms}\frac{R}{Z} = I_{rms}^2 R$$

32.7 Resonance in a Series *RLC* Circuit

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ --> $Z^2 = R^2 + (X_L - X_C)^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$

--> $Z^2 = R^2 + \frac{L^2}{\omega^2}\left(\omega^2 - \frac{1}{LC}\right)^2 = R^2 + \frac{L^2}{\omega^2}\left(\omega^2 - \omega_0^2\right)^2$



The power supplied to the resistor R is $P_{av} = I_{rms}^2 R$.

$$P_{av} = I_{rms}^2 R = \left(\frac{V_{app,rms}}{Z}\right)^2 R = \frac{V_{app,rms}^2}{R^2 + \frac{L^2}{\omega^2}\left(\omega^2 - \omega_0^2\right)^2}R$$

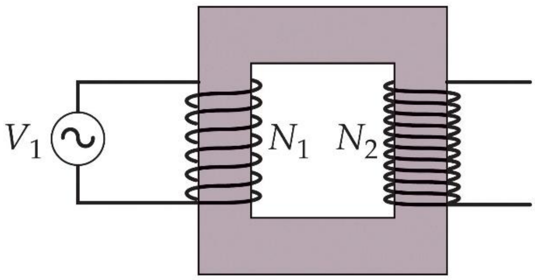
The dissipated power has a maximum value at the resonance frequency

$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$.

Full width at half maximum: $\Delta\omega = R / L$

Quality Factor: $Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R}$

32.8 The Transformer and Power Transmission



Assume that the magnetic flux through a single turn of coil is ϕ_m.

Loop 1: $V_1 - N_1 \frac{d\phi_m}{dt} = 0$ --> $\frac{d\phi_m}{dt} = \frac{V_1}{N_1}$

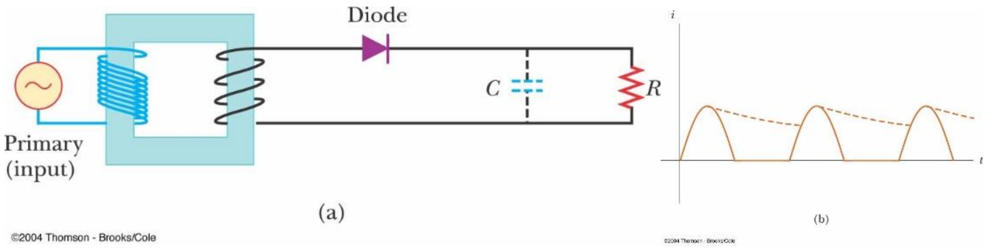
Loop 2: (The coils behave as a voltage source.)

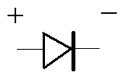
$V_2 = N_2 \frac{d\phi_m}{dt} = \frac{N_2}{N_1} V_1$

Energy conserved: the power input from the generator is equal to the power output,
$P_1 = P_2$ --> $V_1 I_1 = V_2 I_2$.

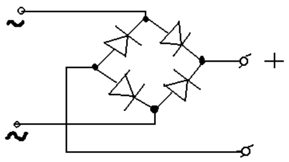
If the input voltage is 110 V and the output voltage is 12 V, which copper wire of the coils is thick? (the input coils or the output coils)

32.9 Rectifiers and Filters

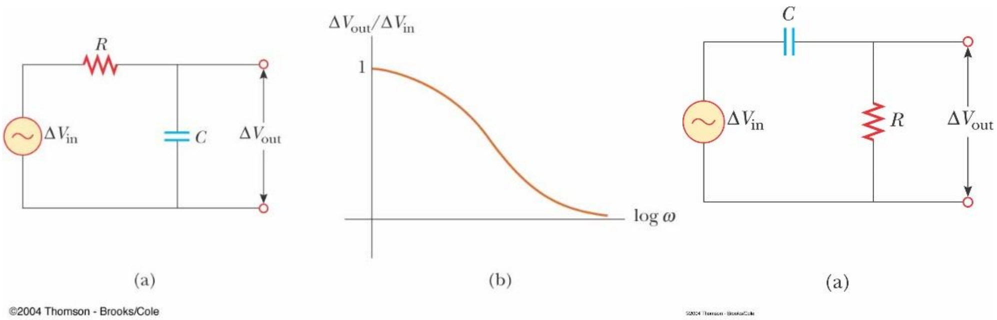




二极管只允许电流
从正极流向负极



四个二极管组成桥式整流电路



Low Pass Filter – High Pass Filter