Chapter 33 Electromagnetic Waves

1. static charge (Ch 21, 22, 23) --> Coulomb and Gauss's law: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

2. static charge --> $\oint \vec{E}\cdot d\vec{l} = 0$

   flux change and Faraday's law (Ch 28) --> $\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_M}{dt}$

3. current generates field, no magnetic monopole --> $\oint \vec{B}\cdot d\vec{A} = 0$

4. static current (Ch 27) --> $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

Lorentz force law (Ch 26): moving charge or current in the magnetic field -->

$\vec{F} = q\vec{v}\times\vec{B} = I\vec{l}\times\vec{B}$

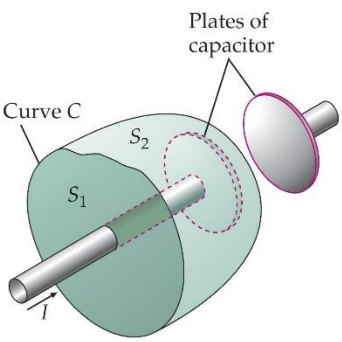
No monopole --> $\oint \vec{B}\cdot d\vec{A} = 0$

Is there any similar formula, like Faraday's law, for the magnetic field?

$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_M}{dt}$ --> $\oint \vec{B}\cdot d\vec{l} \propto \frac{d\Phi}{dt}$

33.1 Displacement Current and the General Form of Ampere's Law

Ampere's Law: $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$

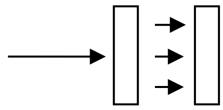


On loop S_1, you enclosed the current I inside, but no current is enclosed on loop S_2. Remember that the loop S_1 can extend the area to the region of zero current. What happens?

The charge (σ) or electric displacement ($\vec{D}$) is changing with time and generating another kind of current: the Maxwell's displacement current $I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$

The generalized Ampere's law: $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$

Derivation 1:



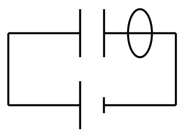
$$I = \frac{dQ}{dt}$$

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} \quad \text{-->} \quad \frac{dQ}{dt} = \varepsilon_0 \frac{d}{dt}\Phi_E \equiv I_d$$

Current I flows in the capacitor --> change the electric field in the capacitor
The change of the electric field seems to be one kind of current:

$$I_d = \varepsilon_0 \frac{d}{dt}\int \overline{E}\cdot d\vec{A}$$

Displacement current density: $\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$



If the capacitor plates are very close, the electric field between them is $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$. A changing electric field may result in a current $\frac{dE}{dt} = \frac{1}{\varepsilon_0}\frac{dQ/dt}{A} = \frac{1}{\varepsilon_0}\vec{J}_d$ → $\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$.

Example: A parallel plate capacitor has closely spaced circular plates of radius R. Charge is flowing onto the positive plate and off the negative plate at the rate $I = \frac{dQ}{dt} = 2.5\text{A}$. Compute the displacement current through surface S passing between the plates by directly computing the rate of change of the flux of $\vec{E}$ through surface S.

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(ES) = \varepsilon_0 \frac{d}{dt}\left(\frac{Q}{S\varepsilon_0}S\right) = \frac{dQ}{dt} = I$$

Example: The circulate plates have a radius of R = 3.0 cm. Find the magnetic field strength B at a point between the plates a distance r = 2.0 cm from the axis of the plates when the current into the positive plate is 2.5 A.

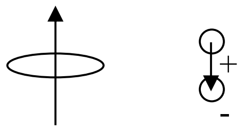
$$\Phi_E = \left(\left(\frac{2}{3}\right)^2 S\right)E = \left(\left(\frac{2}{3}\right)^2 S\right)\frac{Q}{S\varepsilon_0}, \quad I_d = \varepsilon_0 \frac{d}{dt}\Phi_E = \frac{4}{9}\frac{dQ}{dt} = \frac{4I}{9}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_d \text{ --> } 2\pi(0.002)B = \mu_0\left(\frac{4}{9}\right)(2.5A)$$

33.2 Maxwell's Equations and Hertz's Discoveries

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$ Gauss's Law for Electric Fields

$\oint \vec{B}\cdot d\vec{A} = 0$ Gauss's Law for Magnetism

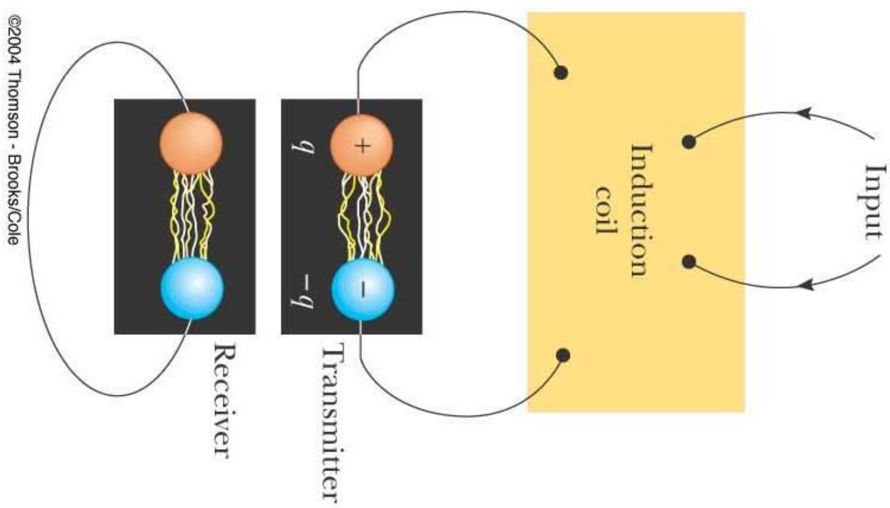


$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$ Faraday's Law

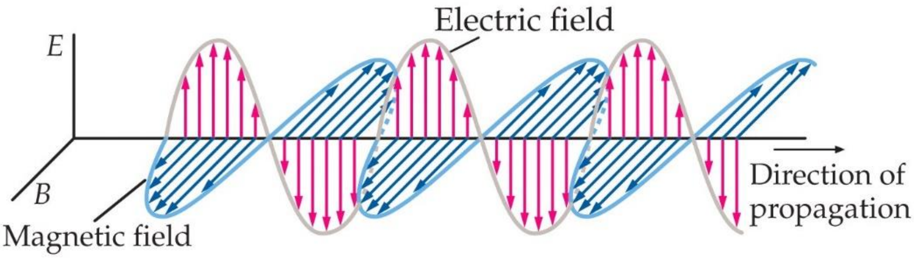
$\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$ Ampere's Law with Maxwell's Displacement Currents

Displacement Current --> Charge Conservation? Symmetry?

$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$ Lorentz Force Law



33.3 Plane Electromagnetic Waves



1. The magnetic and electric fields vary with time and displacement.
2. The electric and magnetic fields are mutually orthogonal.
3. The magnetic field strength B is equal to E/C.
4. The propagating speed is $C = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$.
5. The direction of propagation is $\vec{E} \times \vec{B}$

Electromagnetic Waves:

light, infrared waves, radio-frequency waves

microwave oven (2.45 or 2.5 GHz):
Microwaves are absorbed by water, fats and sugars. When they are absorbed they are converted (through frictional mechanism) into atomic motion - heat. They are not absorbed by most plastics, glass or ceramics. Metal reflects microwaves, this is why metal pans do not work well in a microwave oven.

The Wave Equation for Electromagnetic Waves

How can we get a differential equation of electromagnetic waves from the four fundamental equations for electricity and magnetism?

A propagated plane wave function can be $e^{i(kx-\omega t)}$ or $\sin(kx-\omega t)$. Since $\dfrac{d^2}{dx^2}\sin(kx-\omega t) = -k^2 \sin(kx-\omega t)$ and $\dfrac{d^2}{dt^2}\sin(kx-\omega t) = -\omega^2 \sin(kx-\omega t)$, we may suggest a differential equation of $\left(\dfrac{d^2}{dx^2}\Psi\right)\omega^2 = \left(\dfrac{d^2}{dx^2}\Psi\right)k^2$. --> A differential equation maybe written as $\frac{d^2}{dx^2}\psi = \frac{1}{c^2}\frac{d^2}{dt^2}\psi$.

A changing electric field induced a magnetic field and, at the same time, the varying magnetic field induced an electric field. The induction process maintain the propagation of electromagnetic waves.

What is a plane wave?

What is a spherical wave?

Derivation of the Wave Equation

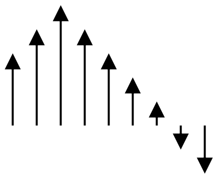
When electromagnetic waves propagate through vacuum space, the current and charge are zero (I = 0 and q = 0). The four fundamental equations (Maxwell's equations) are

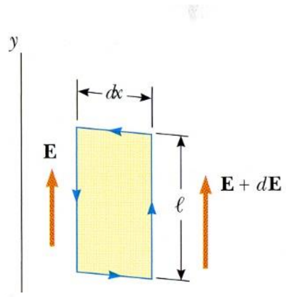
$$\oint \vec{E}\cdot d\vec{A} = 0 \quad \text{--- (1)}$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \text{--- (2)}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \text{--- (3)}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} \quad \text{--- (4)}$$

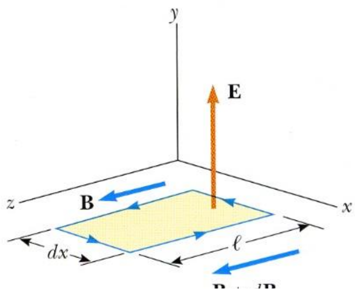




from eq. 3 -> $(E+dE)\cdot l - E\cdot l = -\frac{d}{dt}(B\cdot l\cdot dx)$

$$dE = \frac{\partial E}{\partial x}dx, \quad \frac{\partial E}{\partial x}dx\cdot l = -\frac{dB}{dt}l\cdot dx$$

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \quad \text{-- (important)}} \text{ --- (5)}$$



Serway/Jewett; Principles of Physics, 3/e
Figure 24.6

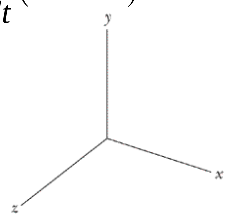
from eq. 4 ->

$$-(B+dB)\cdot l + B\cdot l = \mu\varepsilon\frac{d}{dt}(E\cdot l\cdot dx) \text{->} -dB\cdot l = \mu\varepsilon\frac{d}{dt}(E\cdot l\cdot dx) \ \ \& \ \ dB = \frac{\partial B}{\partial x}dx$$

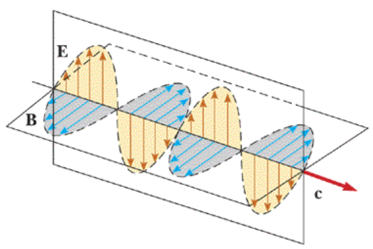
$$\boxed{-\frac{\partial B}{\partial x} = \mu\varepsilon\frac{dE}{dt} \quad \text{-- (important)}} \text{ --- (6)}$$

Eq. (5) & Eq. (6)--> $\frac{\partial^2}{\partial x^2}E = -\frac{\partial}{\partial t}\frac{\partial}{\partial x}B = \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$

$$\frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial x}\right) = \mu\varepsilon\frac{\partial}{\partial x}\frac{\partial E}{\partial t} = -\mu\varepsilon\frac{\partial^2 B}{\partial t^2}$$



$$\vec{E} = E_{\max}\cos(kx-\omega t)\hat{i} \quad \& \quad \vec{B} = B_{\max}\cos(kx-\omega t)\hat{j}$$



Transverse waves

$$\frac{\partial}{\partial x}\left(E = E_{\max}\cos(kx-\omega t)\right) -> \frac{\partial E}{\partial x} = -kE_{\max}\sin(kx-\omega t)$$

$$\frac{\partial}{\partial t}\left(B = B_{\max}\cos(kx-\omega t)\right) -> \frac{\partial B}{\partial t} = \omega B_{\max}\sin(kx-\omega t), \frac{\partial B}{\partial t} = -\frac{\partial E}{\partial x}$$

$$\frac{kE_{\max}}{\omega B_{\max}} = 1, \quad \frac{E_{\max}}{B_{\max}} = c \quad -> \quad \frac{E}{B} = c$$

Example: The electric field of an electromagnetic wave is given by

$\vec{E}(x,t) = E_0\cos(kx-\omega t)\hat{k}$. (a) What is the direction of propagation of the wave? (b) What is the direction of the magnetic field in the x = 0 plane at time t = 0? (c) Find the magnetic field of the same wave. (d) Compute $\vec{E}\times\vec{B}$.

(a) $\hat{i}$

(b) $\hat{k}\times\hat{j} = -\hat{i}$ --> $\hat{k}\times\left(-\hat{j}\right) = \hat{i}$

(c) $\vec{B}(x,t) = \frac{E_0}{c}\cos(kx-\omega t)\left(-\hat{j}\right)$

(d) $\vec{E}\times\vec{B} = \frac{E_0^2}{c}\cos^2(kx-\omega t)\hat{i}$

Example: The electric field of an electromagnetic wave is given by

$\vec{E}(x,t) = \hat{j}E_0\sin(kx-\omega t) + \hat{k}E_0\cos(kx-\omega t)$. (a) Find the magnetic field of the same wave. (b) Compute $\vec{E}\cdot\vec{B}$ and $\vec{E}\times\vec{B}$.

(a) The propagation direction is in the x-direction $\hat{i}$.

$\hat{j}\times\hat{k} = \hat{i}$ & $\hat{k}\times\left(-\hat{j}\right) = \hat{i}$

$$\vec{B}(x,t) = \hat{k}\frac{E_0}{c}\sin(kx-\omega t) + \left(-\hat{j}\right)\frac{E_0}{c}\cos(kx-\omega t)$$

(b) $\vec{E}\cdot\vec{B} = 0$, $\vec{E}\times\vec{B} = \hat{i}\frac{E_0^2}{c}$

<u>33.4 Energy Carried by Electromagnetic</u>

Waves

Intensity:

$$I = \frac{P_{av}}{A} = \frac{U_{av}/\Delta t}{A} = \frac{u_{av}V/\Delta t}{A} = u_{av}\frac{L}{\Delta t} = u_{av}c$$

$$u_{av} = u_e + u_m$$

$$u_e = \frac{1}{2}\varepsilon_0 E^2,\ u_m = \frac{B^2}{2\mu_0} = \frac{E^2}{2\mu_0 c^2} = \frac{\varepsilon_0 E^2}{2} \text{ --> } u_{av} = u_e + u_m = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} = \frac{EB}{\mu_0 c}$$

$$I = u_{av}c = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_0 B_0}{2\mu_0} = \left|\vec{S}\right|_{av}$$

Poynting Vector:

$\vec{S} = \dfrac{\vec{E}\times\vec{B}}{\mu_0}$ a direction in which the energy is transmitted?

Example: Fields due to a point source

A point source of em radiation has an average power output of 800 W. Calculate the maximum values of the electric and magnetic fields at a point 3.5 m from the source.

$$I = \frac{P}{4\pi r^2} = \frac{800}{4\pi\cdot 3.5^2} = 5.2W/m^2$$

$$E_{\max} = \sqrt{2\mu_0 cI} = 62.6V/m,$$

33.5 Momentum and Radiation Pressure

Charged particle experience electric fields of the EM Waves:

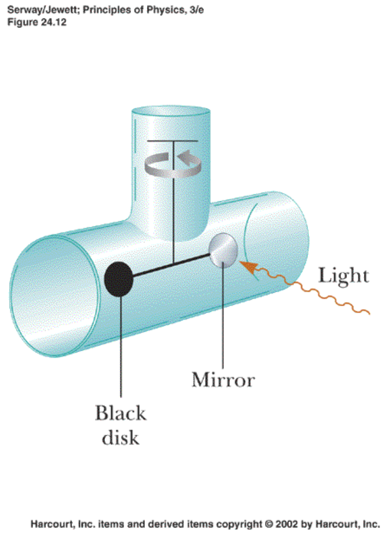
$$v_y = at = \frac{qE}{m}t \text{ --> } K = \frac{q^2E^2}{2m}t^2$$

$$F_B = qvB = q\left(\frac{qE}{m}t\right)B$$

the transferred momentum in time t_1: $p = \int F dt = \frac{q^2EB}{m}\frac{t_1^2}{2}$

$$\text{-->}\ p = \frac{1}{c}\frac{q^2E^2}{2m}t^2 = \frac{K}{c}$$

momentum change when adsorbing em waves: $p = \frac{U}{c}$

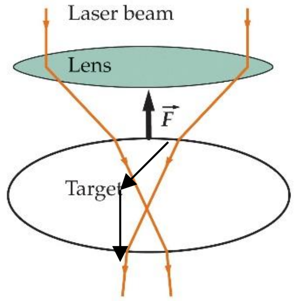


if the intensity is I and the pressure is P

$$P = \frac{F}{A} = \frac{1}{A}\frac{p}{t} = \frac{1}{At}\frac{U_{av}}{c} = \frac{U_{av}}{Al}\frac{l}{t}\frac{1}{c} = u_{av} \cdot c \cdot \frac{1}{c} = \frac{I}{c}$$

for a perfect reflector: $P = 2\frac{I}{c}$

Laser Tweezer:



Example: Solar energy

The sun delivers 1000 W/m^2 of energy to the Earth's surface. (a) Calculate the total power incident on a roof of dimension 8 m x 20 m.

Power: $P = 1000 \times 8 \times 20 = 1.6 \times 10^5 W$

Example: Pressure from a laser pointer

If a 3mW pointer creates a spot with a diameter of 2 mm. Determine the radiation pressure on a screen that reflects 70% of the light striking it.

Intensity: $I = \frac{3 \times 10^{-3}}{\pi \left(10^{-3}\right)^2} = 9.6 \times 10^2 W/m^2$

Pressure: $P = \frac{I}{c}(1 + 0.7) = 5.4 \times 10^{-6} N/m^2$

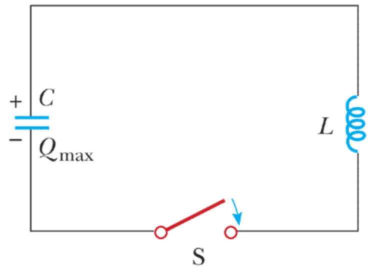
Space Sailing by Using the Radiation Pressure?

Example: You are stranded in space a distance of *s* from your spaceship. You carry a laser with power P_{av}. If your total mass, including your space suit and laser, is *m*, how long will it take you to reach the spaceship if you point the laser directly away from it?

$$P_{av} = \frac{dU}{dt}, \quad F = \frac{dp}{dt} = \frac{d}{dt}\left(\frac{U}{c}\right) = \frac{1}{c}\frac{dU}{dt} = \frac{1}{c}P_{av} = ma$$

--> $a = \frac{P_{av}}{mc}$ & $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2smc}{P_{av}}}$

33.6 Production of Electromagnetic Waves by an Antena

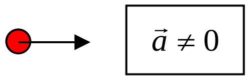


$$-\frac{Q}{C}-L\frac{dI}{dt}=0$$

$$Q+LC\frac{d^2Q}{dt^2}=0\,,\quad \frac{d^2Q}{dt^2}+\frac{Q}{LC}=0$$

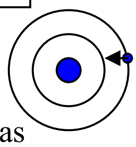
$$F=m\frac{d^2x}{dt^2}=-kx\,,\quad x=x_0\cos(\omega t),\quad \omega=\sqrt{\frac{k}{m}}$$

$$\frac{d^2Q}{dt^2}+\frac{Q}{LC}=0\,,\quad Q=Q_0\cos(\omega t),\quad \omega=\frac{1}{\sqrt{LC}}$$

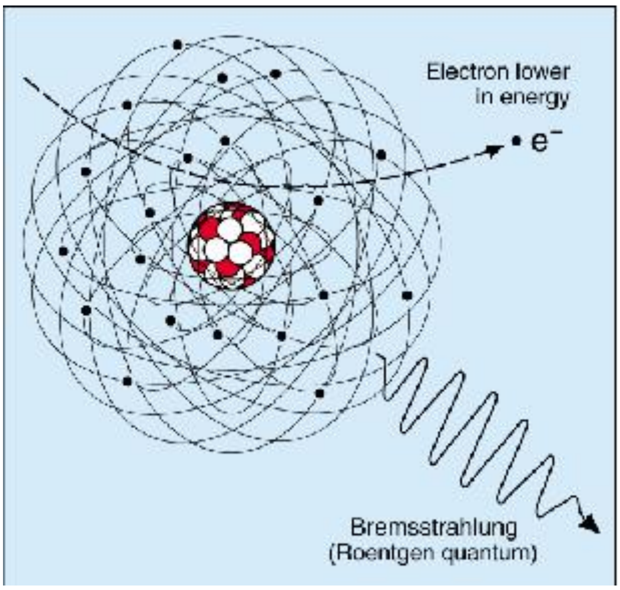


Generating EM Waves:

1. accelerate or decelerate free charges
2. light wave: when outer electrons bounded to atoms make transitions
3. macroscopic electric currents oscillating in radio transmission antennas
4. vibration --> infrared EM waves

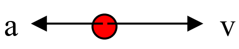


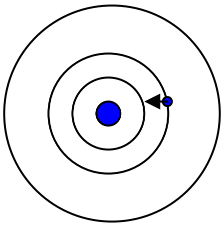
deceleration of electrons when crashing into a metal target --> bremsstrahlung



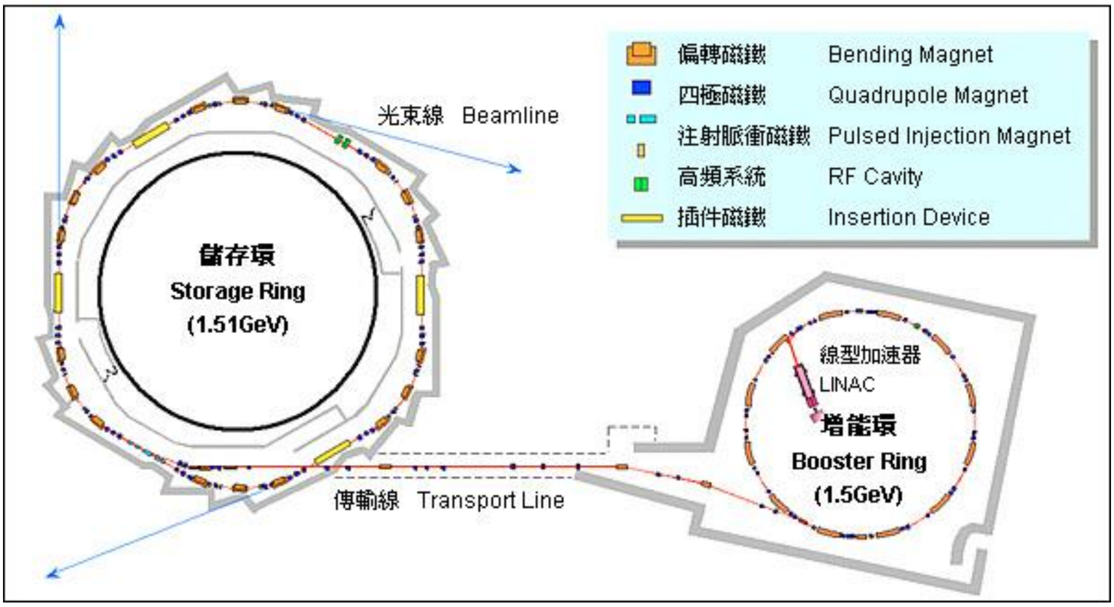
Ref: http://www.ghettodriveby.com/bremsstrahlung/

X-Ray:



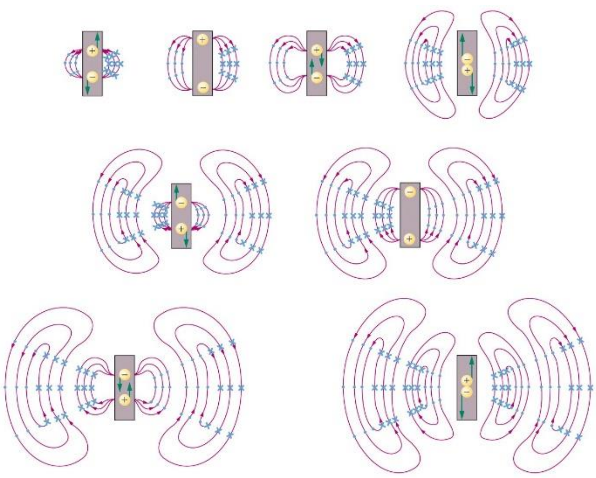


Synchrotron Radiation:



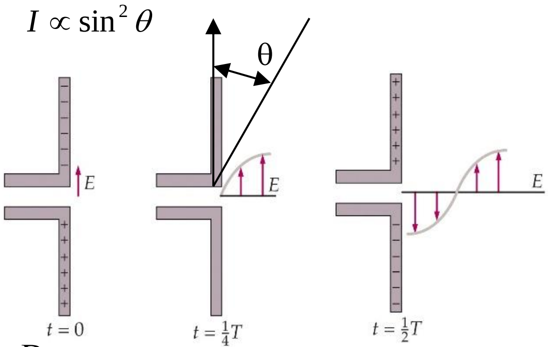
Electric Dipole Radiation

Generator:

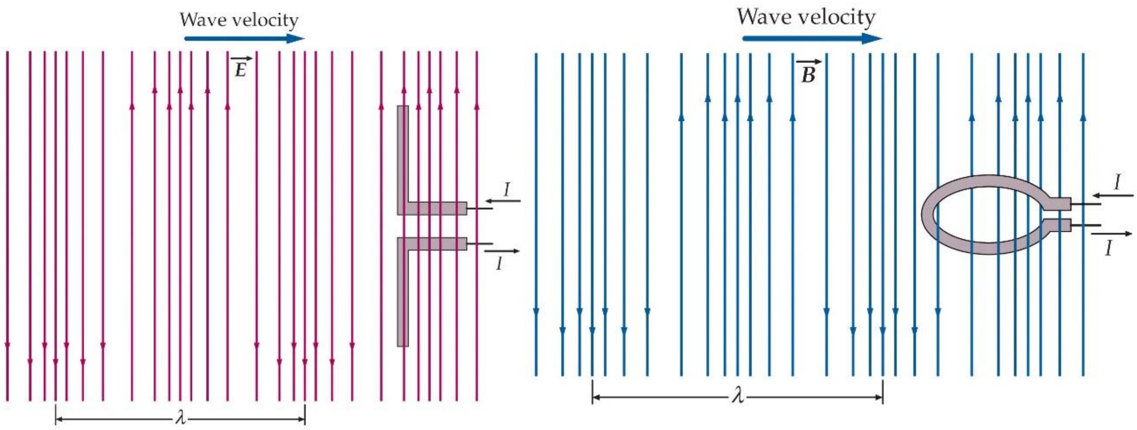


Electric Dipole Radiation

$I \propto \sin^2\theta$



Detector:



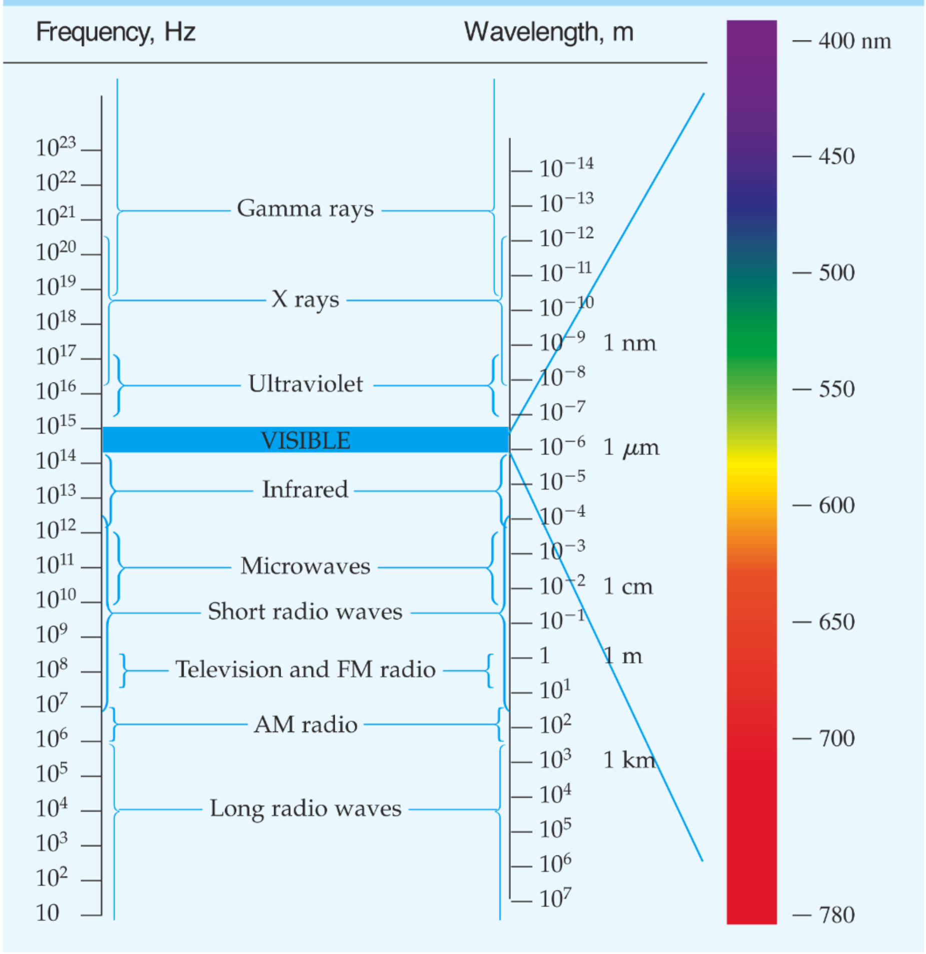
Example: A loop antenna consisting of a single 10-cm radius loop of wire is used to detect electromagnetic waves for which $E_{rms} = 0.15$ V/m. Find the rms emf induced in the loop if the wave frequency is (a) 600 kHz and (b) 60 MHz.

$$B_{rms} = \frac{E_{rms}}{c}, \quad \varepsilon = -\frac{d}{dt}(BA) \;\text{-->}\; \varepsilon_{rms} = \omega B_{rms}\left(\pi r^2\right) = \omega \frac{E_{rms}}{c}\left(\pi r^2\right)$$

33.7 The Spectrum of Electromagnetic Waves

TABLE 30-1

The Electromagnetic Spectrum



Radio waves, Microwaves, Infrared waves, Visible light, Ultraviolet waves, X-rays, Gamma rays

Visible Light: $\lambda = 400$ nm --> $f = \dfrac{3\times10^{8}}{400\times10^{-9}} = 7.5\times10^{14}$ Hz -->

$E = hf = \left(6.626\times10^{-34}\right)\left(7.5\times10^{14}\right)/\left(1.602\times10^{-19}\right) = 3.1$ eV

X-Ray: $\lambda = 0.1$ nm --> $f = \dfrac{3\times10^{8}}{0.1\times10^{-9}} = 3\times10^{18}$ Hz -->

$E = hf = \left(6.626\times10^{-34}\right)\left(3\times10^{18}\right)/\left(1.602\times10^{-19}\right) = 12.4$ keV