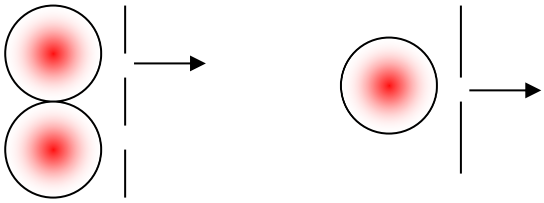
Chapter 36 Interference of Light

Waves



Interference and Diffraction – Important phenomena that distinguish waves from particles

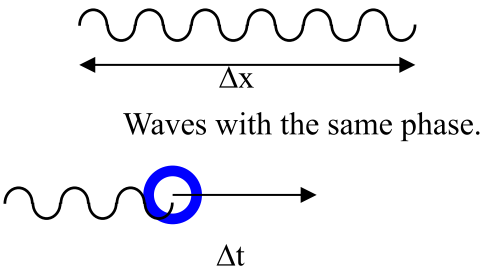
Diffraction – Bending of waves around corners

36.1 Conditions for Interference

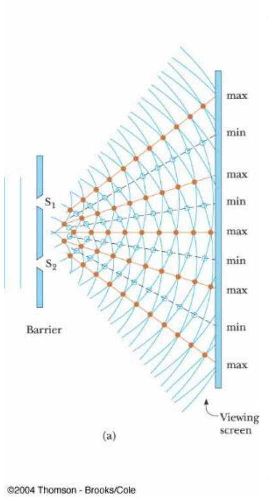
1. The source must be coherent; that is, they must maintain a constant phase with respect to each other.
2. The sources should be monochromatic; that is, they should be of a single wavelength.

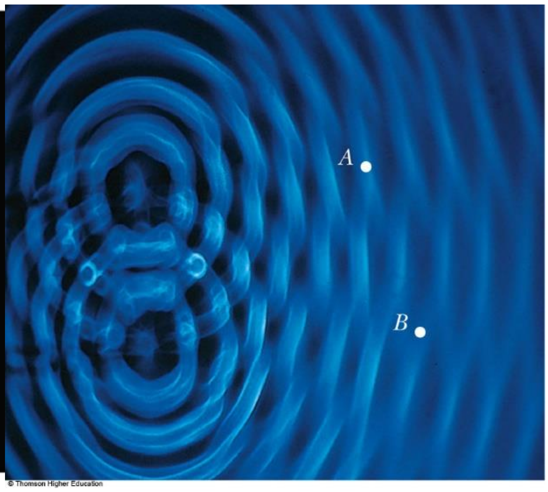
What's Phase Coherence?

What is the coherence length of the light?
What is the coherent time of the light?



36.2 Young's Double-Split Experiment





36.3 Light Waves in Interference

Phase Difference

Waves: $A\cos(kx-\omega t+\delta)$

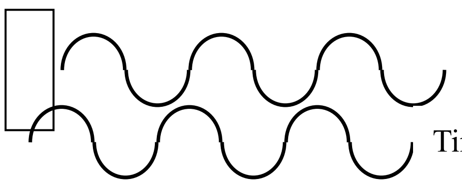
Two harmonic waves of the same frequency and wavelength, the resultant wave is a harmonic wave: $A_1\sin(kx-\omega t)+A_1\sin(kx-\omega t+\delta)=2A_1\cos(\delta/2)\sin(kx-\omega t+\delta/2)$.

phase difference is zero --> in phase, interfere constructively

phase difference is 180º --> out of phase, interfere destructively

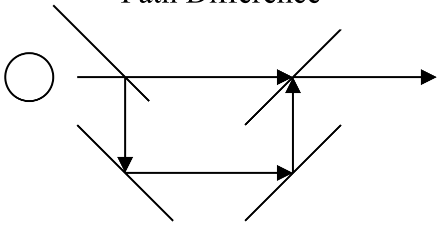
phase difference: δ, can be a path difference or a time delay

$\delta = 2\pi\frac{\Delta x}{\lambda} = k\Delta x$ or $\delta = 2\pi\frac{\Delta t}{T} = \omega\Delta t$



Time Delay

Path Difference

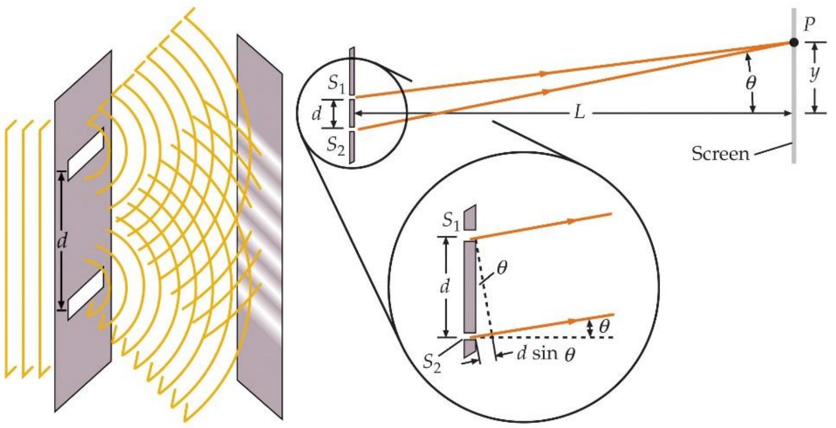


Example: (a) What is the minimum path difference that will produce a phase difference of 180° for light of wavelength 800 nm? (b) What phase difference will that path difference produce in light of wavelength 700 nm?

(a) $\frac{\pi}{2\pi} = \frac{\Delta x}{800}$ --> $\Delta x = 400$ nm

(b) $\delta = 2\pi \frac{400}{700}$

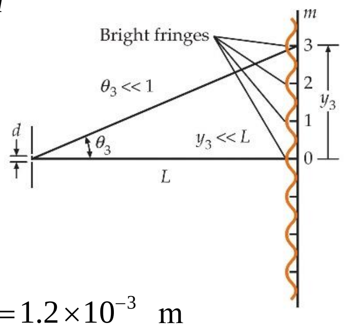
Simple Explanation of Double-Slit Interferene



Constructive: $d\sin\theta = m\lambda$, Destructive: $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$

The phase difference between lights from the two slits is: $\delta = 2\pi \frac{d\sin\theta}{\lambda}$

$\tan\theta = \frac{y}{L}$ --> $y_{\max imum\_\mathrm{int}\, ensity} = L\tan\theta_m \sim L\sin\theta_m \sim L\frac{m\lambda}{d}$



Example: Two narrow slits separated by 1.5 mm are illuminated by yellow light of wavelength 589 nm from a sodium lamp. Find the spacing of the bright fringes observed on a screen 3 m away.

$y_m = L\tan\theta \sim L\sin\theta = L\frac{m\lambda}{d}$ --> $y_1 = L\frac{\lambda}{d} = 3\frac{589}{1.5\times10^6} = 1.2\times10^{-3}$ m

Example: Measuring the wavelength of laser light
A laser is used to illuminate a double slit. The distance between the two slit is 0.03 mm. A viewing screen is separated from the double slits by 1.2 m. The second-order brighter fringe m = 2 is 5.1 cm from the center line. (a) Determine the wavelength of the laser light.

$$y_{bright} = m\frac{L\lambda}{d} \quad \& \quad m = 2 \quad \text{-->} \quad 5.1\times 10 = 2\frac{\left(1.2\times 10^{3}\right)\lambda}{0.03} \quad \text{-->} \quad \lambda = 6.375\times 10^{-4} \ \text{ mm}$$

36.4 Intensity Distribution of the Double-Slit Interference Pattern

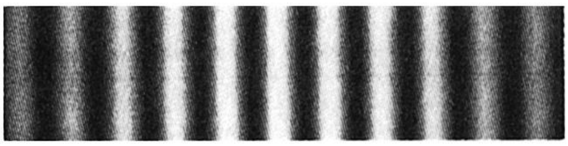
$$\psi_1 = A\sin(kx-\omega t),\ \ \psi_2 = A\sin(kx-\omega t+\delta)$$

$$I_0 = I_1 + I_2 = \left\langle A^2\sin^2(kx-\omega t)\right\rangle + \left\langle A^2\sin^2(kx-\omega t+\delta)\right\rangle = \frac{A^2}{2}+\frac{A^2}{2} = A^2$$

$$\psi' = A\sin(kx-\omega t) + A\sin(kx-\omega t+\delta) = 2A\cos(\delta/2)\sin(kx-\omega t+\delta/2)$$

$$I = \left\langle \psi^2\right\rangle = \left\langle 4A^2\cos^2\left(\frac{\delta}{2}\right)\sin^2\left(kx-\omega t+\frac{\delta}{2}\right)\right\rangle$$

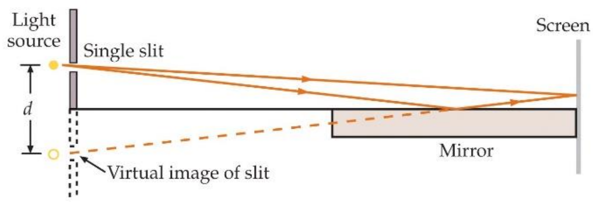
$$I = \left\langle \psi^2\right\rangle = 4A^2\cos^2\left(\frac{\delta}{2}\right)\frac{1}{2} = 2A^2\cos^2\left(\frac{\delta}{2}\right)$$



$$\delta = k(\Delta x) = kd\sin\theta \sim kd\tan\theta = kd\frac{y}{L}$$

$$\text{-->} \quad I = 2A^2\cos^2\left(\frac{\pi dy}{\lambda L}\right) \quad \text{or} \quad I = 2I_0\cos^2\left(\frac{\delta}{2}\right)$$

Lloyd's mirror for producing a two-slit interference pattern

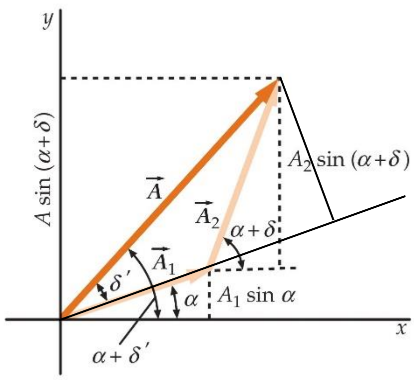


Phasor for Wave Addition

$$\alpha = kx - \omega t$$

$$E = E_1 + E_2 = A_1\cos(\alpha) + A_2\cos(\alpha+\delta)$$

$$A^2 = A_1^2 + A_2^2 - 2A_1A_2\cos(\pi-\delta)$$

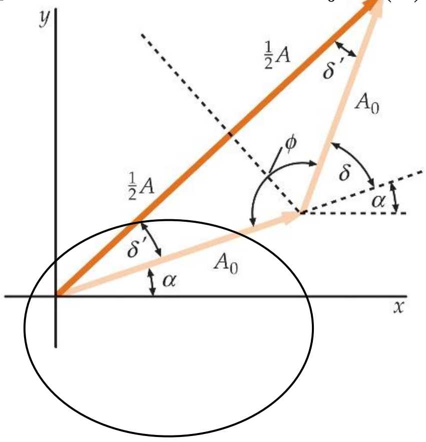


$$\tan(\alpha+\delta')=\frac{A_1\sin\alpha+A_2\sin(\alpha+\delta)}{A_1\cos\alpha+A_2\cos(\alpha+\delta)}$$

$$\tan\delta'=\frac{A_2\sin\delta}{A_1+A_2\cos\delta}$$

--> $E=A\cos(\alpha+\delta')=A\cos(kx-\omega t+\delta')$

Example: Use the phasor method to derive the superposition of two waves, $A_0\sin(\alpha)$ and $A_0\sin(\alpha+\delta)$, of the same amplitude.



$$\delta'=\frac{\delta}{2}$$

$$\frac{A}{2}=A_0\cos\delta'=A_0\cos\left(\frac{\delta}{2}\right)$$

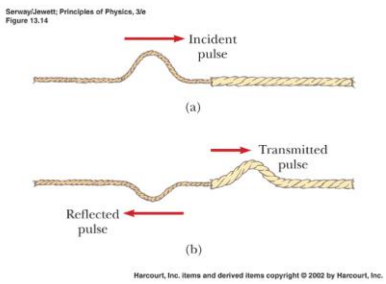
--> $E=A\sin(\alpha+\delta')=2A_0\cos\left(\frac{\delta}{2}\right)\sin\left(\alpha+\frac{\delta}{2}\right)$

36.5 Change of Phase Due to Reflection

If light traveling in one medium strikes the surface of a medium in which light travels more slowly, there is a 180° phase change in the reflected light.

As light travel from one medium to another, its frequency does not change but its wavelength does.

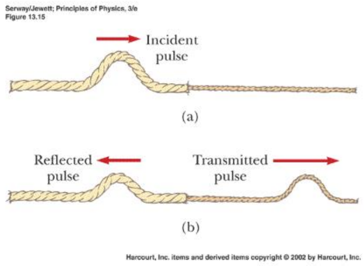
Relative Intensity of Reflected and Transmitted Light



$y_l(x,t)=A\cos(k_l x-\omega t)$, $v_l=\sqrt{\frac{T}{\mu_l}}=\frac{\omega}{k_l}$,

$y_l'(x,t)=B\cos(-k_l x-\omega t)$

$y_r(x,t) = C\cos(k_r x - \omega t)$, $v_r = \sqrt{\dfrac{T}{\mu_r}} = \dfrac{\omega}{k_r}$



$x = 0$, $y_l + y_l' = y_r$, ==> $A + B = C$

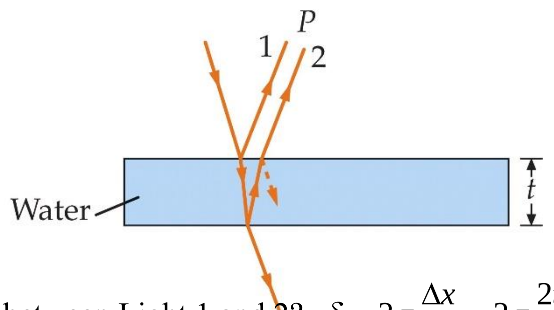
$\dfrac{d}{dx} y_l + \dfrac{d}{dx} y_l' = \dfrac{d}{dx} y_r$, ==> $k_l(A - B) = k_r C$

$$B = \frac{k_l - k_r}{k_l + k_r} A = \frac{n_l - n_r}{n_l + n_r} A, \quad C = \frac{2k_l}{k_l + k_r} A = \frac{2n_l}{n_l + n_r} A$$

$n_l > n_r$ --> B > 0 (in phase) : $n_l < n_r$ --> B < 0 (out of phase)

The reflected intensity can be $I = I_0 \left(\dfrac{B}{A}\right)^2 = \left(\dfrac{n_l - n_r}{n_l + n_r}\right)^2 I_0$

<u>36.6 Interference in Thin Films</u>



Coherent <-- The same source light

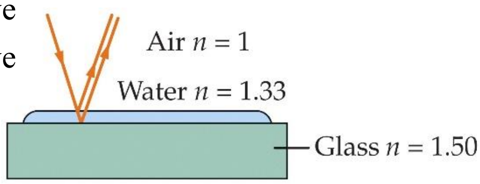
Light 1: 180° (π) reflected wave

Light 2: 0° reflected wave

Constructive: $\delta = \pi$

Destructive: $\delta = 2n\pi$

How do we estimate the phase difference between Light 1 and 2? $\delta = 2\pi \dfrac{\Delta x}{\lambda} \sim 2\pi \dfrac{2t}{\lambda}$



Light 1: 180° (π) reflected wave

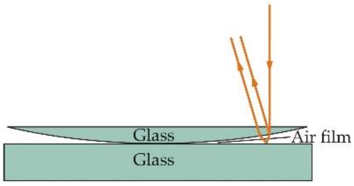
Light 2: 180° (π) reflected wave

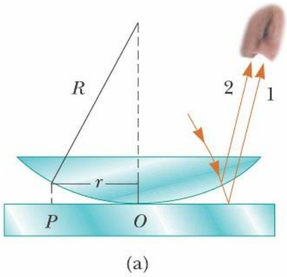
$\delta = 2\pi \dfrac{\Delta x}{\lambda} \sim 2\pi \dfrac{2t}{\lambda}$

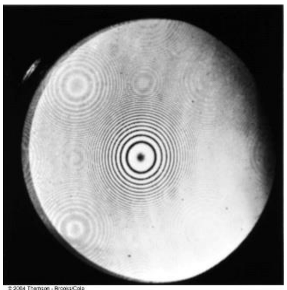
Constructive: $\delta = 2n\pi$

Destructive: $\delta = \pi$

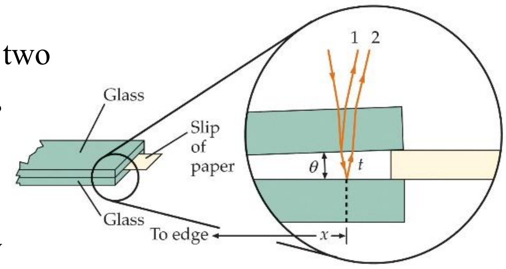
Newton's rings: the thin film is air







Example: A wedge-shaped film of air is made by placing a small slip of paper between the edges of two flat pieces of glass. Light of wavelength 500 nm is incident normally on the glass, and interference fringes are observed by reflection. If the angle θ made by the two plates is 3×10^{-4} rad, how many dark interference fringes per centimeter are observed?



Number of rings: $n = \dfrac{2t}{\lambda}$, how to estimate t?

Using small angle approximation: $\dfrac{t}{L} \sim \theta$ → $n = \dfrac{2\theta}{\lambda}L$ → number of rings per unit length $\dfrac{n}{L} = \dfrac{2\theta}{\lambda}$, $\dfrac{n}{L} = \dfrac{2\left(3\times10^{-4}\right)}{5\times10^{-7}} = 12$ cm^{-1}

36.7 The Michelson Interferometer