Chapter 33
Electromagnetic Waves

Physics II – Part II
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Discovery of Charges

History

1827 AD – Joseph Henry (American, 1797-1878)

Discovered self-inductance & mutual-inductance.
Invention of the electromagnet.
In 1831, he built an electric motor – DC motor.

1864 AD – James Clerk Maxwell (Scottish, 1831-1879)

In 1865, he published "A Dynamical Theory of the Electromagnetic Field".
Demonstrated the electric and magnetic fields moving in space as waves at the speed of light.

1887 AD – Heinrich Rudolf Hertz (German, 1857-1894)

Experimentally proved the existence of electromagnetic waves in space.
Established the photoelectric effect (later explained by Albert Einstein).

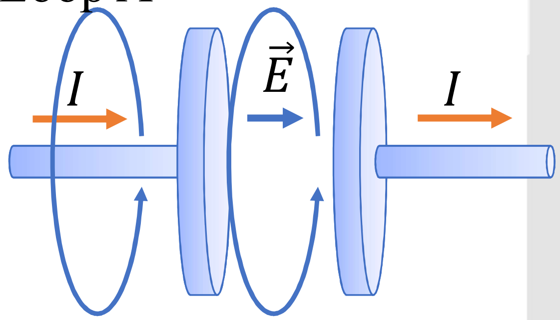
1891 AD – Nikola Tesla (Serbian-American, 1856-1943)

Invented an electrical resonant transformer circuit – tesla coil.

wireless charging pad

Ref: https://edisontechcenter.org/JosephHenry.html, https://en.wikipedia.org/wiki/Heinrich_Hertz, Displacement Current

Maxwell's Correction to Ampere's Law



On Loop A, there are magnetic fields that satisfy the Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

On Loop B, we find the same magnetic fields. Where are current sources?

We observe a change of electric field on Loop B. The electric flux gives us a quantity of charge thus the change of electric flux may give us current.

$$Q \propto \varepsilon_0 \iint \vec{E} \cdot d\vec{A} \rightarrow I = \varepsilon_0 \frac{d\Phi_E}{dt}$$

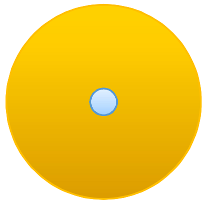
$$I_d = \varepsilon_0 \frac{d}{dt}\left(\iint \vec{E} \cdot d\vec{A}\right) \qquad \vec{J}_d = \varepsilon_0 \frac{d\vec{E}}{dt} \rightarrow \vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Ref:

Maxwell's Equation & Hertz's Discovery

Electromagnetic Waves

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$



Gauss's law for electric fields

$$\oiint \vec{B} \cdot d\vec{A} = 0$$

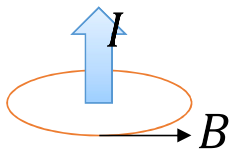
Gauss's law for magnetic fields

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \iint \vec{B} \cdot d\vec{A}$$

Ampere's law for electric fields

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \left(\varepsilon_0 \frac{\partial}{\partial t} \iint \vec{E} \cdot d\vec{A} \right)$$

Ampere's law for magnetic fields

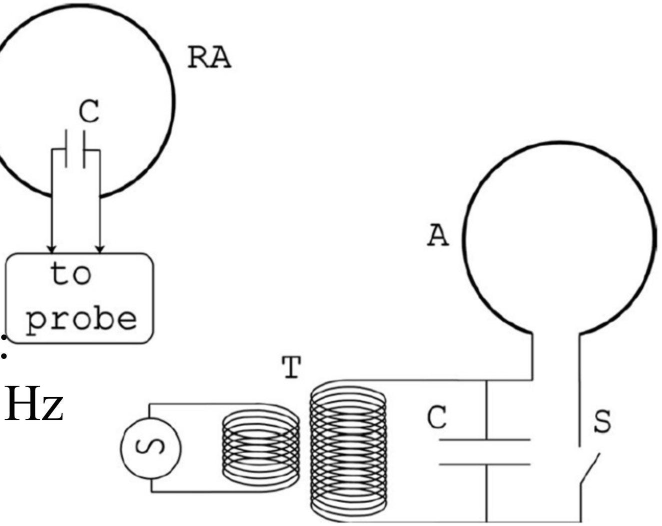


Lorentz force law: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Ref:

Hertz Experiments

Daniele Faccio, Matteo Clerici, and Davide Tambuchi, "Revisiting the 1888 Hertz experiment", Am. J. Phys. 74(11), 992 (2006).



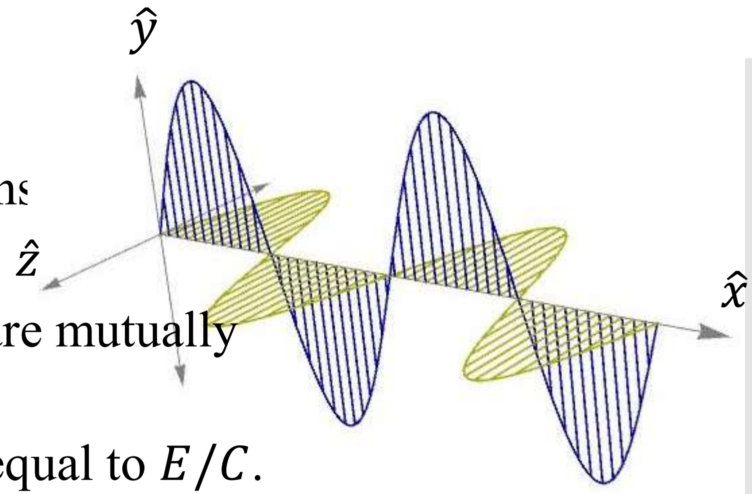
A: emitting antenna, C: 1 nF capacitor, S: spark switch, T: 220 V/50 Hz to 6 kV/50 Hz transformer, RA: receiving antena

When a voltage difference of 6 kV is reached, the switch S is closed and the RLC circuit start to oscillate at a frequency of $\frac{1}{2\pi\sqrt{LC}}$ ~2 MHz. At each oscillation, the resistor of the antenna A will consume 30% of power to generate EM waves. These waves may be captured by the receiver antenna.

Ref:

Plane Waves

1. The electric and magnetic fields change simultaneously as functions time t and displacement x.
2. The electric and magnetic fields are mutually orthogonal.
3. The magnetic field strength B is equal to E/C.
4. The propagation speed of the EM waves is $C = 1/\sqrt{\varepsilon_0 \mu_0}$.
5. The propagation is along the direction of $\vec{E} \times \vec{B}$.



$$\vec{E}(x,t) = E_0 \sin(kx - \omega t)\,\hat{\jmath}\ \&\ \vec{B}(x,t) = B_0 \sin(kx - \omega t)\,\hat{k}, B_0 = E_0/C$$

Light, infrared waves (IR), radio-frequency waves, and microwaves are EM waves.
Microwaves are absorbed by water, fats, and sugar. They cannot be observed plastic, glass, and ceramics while they are reflected by metals.
The EM wave energy are converted to atomic motion, showing heat.

Maxwell's Equations Without Charges and Currents

Electromagnetic Waves in Vacuum

$$\oiint \vec{E}\cdot d\vec{A} = 0 \qquad \oiint \vec{B}\cdot d\vec{A} = 0$$

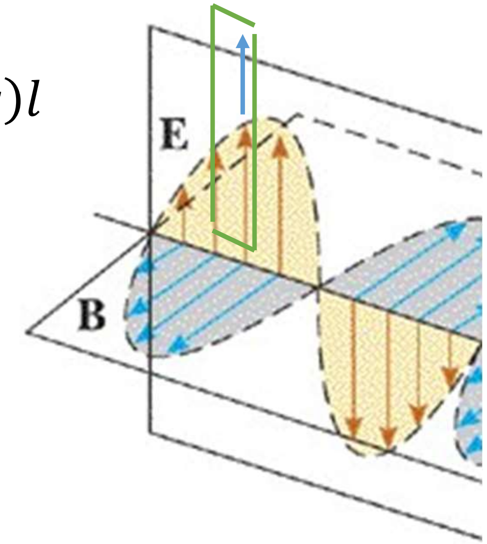
$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot d\vec{A} \qquad \oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{\partial}{\partial t}\iint \vec{E}\cdot d\vec{A}$$

$$0\times dx + E(x+dx)l - 0\times dx - E(x)l$$

$$= -\frac{\partial}{\partial t}(l(dx)B\,)$$

$$\frac{E(x+dx)-E(x)}{dx} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$



Ref:

Maxwell's Equations Without Charges and Currents

Electromagnetic Waves in Vacuum

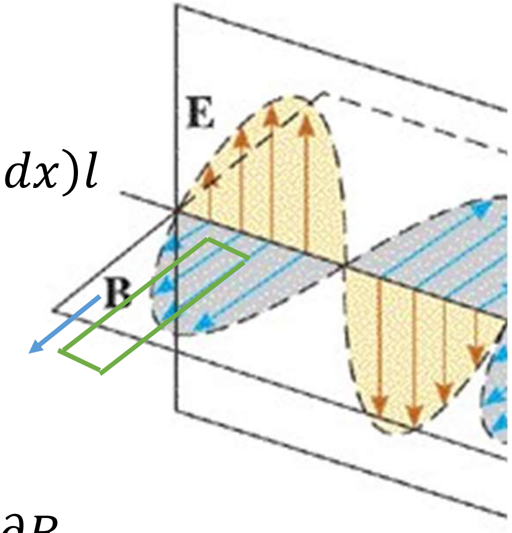
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \iint \vec{E} \cdot d\vec{A}$$

$$0 \times dx + B(x)l - 0 \times dx - B(x+dx)l$$

$$= \mu_0 \varepsilon_0 \frac{\partial}{\partial t} (l(dx)E)$$

$$-\frac{B(x+dx) - B(x)}{dx} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$-\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \qquad \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$



$$-\frac{\partial}{\partial t} \frac{\partial B}{\partial x} = -\frac{\partial}{\partial x} \frac{\partial B}{\partial t} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial}{\partial x} \frac{\partial E}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad \frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = \frac{1}{C^2} \frac{\partial^2 E}{\partial t^2}$$

Ref:

Plane Waves

Electromagnetic Waves in Vacuum

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = \frac{1}{C^2}\frac{\partial^2 E}{\partial t^2} \rightarrow C = 1/\sqrt{\mu_0 \varepsilon_0}$$

Electric & magnetic fields of a **plane wave**:

$$\vec{E}(x,t) = E_0 \sin(kx - \omega t)\,\hat{\jmath} \qquad \vec{B}(x,t) = B_0 \sin(kx - \omega t)\,\hat{k}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \rightarrow kE_0 = \omega B_0 \rightarrow B_0 = \frac{E_0}{C}$$

The propagation direction is $\vec{E} \times \vec{B} \rightarrow \hat{\jmath} \times \hat{k} = \hat{\imath}$.

The wave equation: $\frac{\partial^2}{\partial x^2} E_0 \sin(kx - \omega t) = -k^2 E_0 \sin(kx - \omega t)$

$$\frac{\partial^2}{\partial t^2} E_0 \sin(kx - \omega t) = -\omega^2 E_0 \sin(kx - \omega t)$$

$$\frac{\partial^2}{\partial x^2} E_0 \sin(kx - \omega t) = \frac{k^2}{\omega^2}\frac{\partial^2}{\partial t^2} E_0 \sin(kx - \omega t) \qquad \frac{\partial^2 E}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 E}{\partial t^2}$$

Ref:

Wave Equations

General form of wave equations: $\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$

The electric field satisfies the wave equation:

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 E(x,t)}{\partial t^2}$$

The same as the magnetic field: $\vec{B}(x,t) = B_0 \sin(kx - \omega t)\,\hat{k}$

$$\frac{\partial^2 B}{\partial x^2} = -k^2 B_0 \sin(kx - \omega t) = \frac{k^2}{\omega^2}\left(-\omega^2 B_0 \sin(kx - \omega t)\right) = \frac{k^2}{\omega^2}\frac{\partial^2 B}{\partial t^2}$$

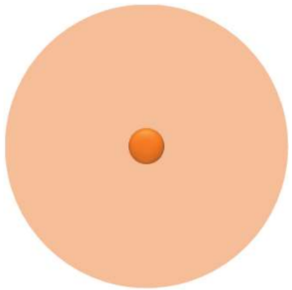
Alternative wave functions:

$$\Psi(x,t) = A_0 e^{i(kx-\omega t)} \rightarrow \frac{\partial \Psi}{\partial x} = ikA_0 e^{i(kx-\omega t)} \rightarrow \frac{\partial^2 \Psi}{\partial x^2} = -k^2 A_0 e^{i(kx-\omega t)}$$

$$\frac{\partial \Psi}{\partial t} = -i\omega A_0 e^{i(kx-\omega t)} \rightarrow \frac{\partial^2 \Psi}{\partial t^2} = -\omega^2 A_0 e^{i(kx-\omega t)} \rightarrow \frac{\partial^2 \Psi}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 \Psi}{\partial t^2}$$

Light Intensity and Energy Density of EM Waves

Light intensity I of a point light source:



$I \propto P, P$ is the power of the light source

$I \propto 1/r^2, r$ is the distance away from the light source

$$4\pi r^2 I = P \rightarrow I = \frac{P}{4\pi r^2} \rightarrow I = \frac{P}{A}$$

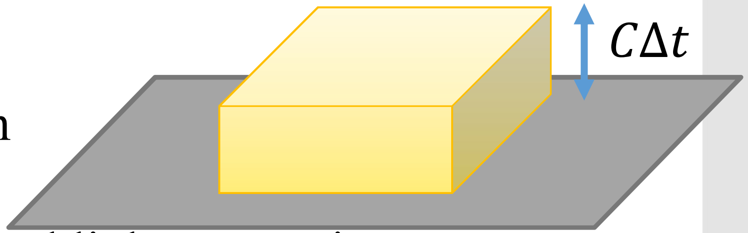
Energy density (energy per unit volume) of the EM wave:

$$u_{EM} = u_E + u_M = \left\langle \frac{\varepsilon_0}{2} E_0^2 \sin^2(kx - \omega t) \right\rangle + \left\langle \frac{1}{2\mu_0} B_0^2 \sin^2(kx - \omega t) \right\rangle$$

$$u_{EM} = \frac{\varepsilon_0}{2} E_0^2 \frac{1}{2} + \frac{1}{2\mu_0} B_0^2 \frac{1}{2} = \frac{1}{2} \varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0} B_{rms}^2$$

$$B_{rms} = \frac{E_{rms}}{c} \rightarrow u_{EM} = \varepsilon_0 E_{rms}^2 = \frac{\varepsilon_0 E_0^2}{2}$$

Light Intensity and Energy Density of EM Waves



In a time duration of Δt, a volume of light is absorbed on the surface with an area A.

The volume of light is $AC\Delta t$ and the total light energy is

$$K = (AC\Delta t)(u_{EM}) = AC\Delta t \frac{\varepsilon_0 E_0^2}{2}$$

The intensity of the light on the surface is

$$I = \frac{(AC\Delta t)(u_{EM})}{\mathrm{A\Delta t}} = Cu_{EM} = \frac{\varepsilon_0 C E_0^2}{2} = \varepsilon_0 C E_{rms}^2$$

The intensity of the light is the energy carried away via wave propagation:

$$I = \varepsilon_0 C E_{rms}^2 = \varepsilon_0 C^2 \frac{E_{rms}^2}{C} = \frac{E_{rms}^2}{\mu_0 C} = \frac{E_{rms} B_{rms}}{\mu_0}$$

The Poynting vector is defined as $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$, thus $\boldsymbol{I = \langle \vec{S} \rangle = Cu_{EM}}$.

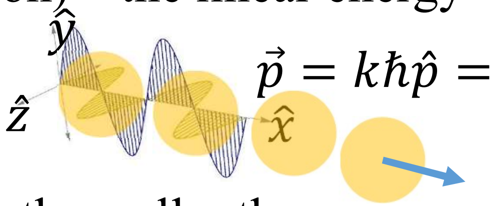
Momentum and Radiation Pressure

The momentum of a classical particle – the parabolic energy (kinetic energy) momentum relation:

$$p = mv, E = \frac{mv^2}{2} \rightarrow E = \frac{p^2}{2m}$$

The momentum of a light quantum (photon) – the linear energy momentum relation: $E = Cp$

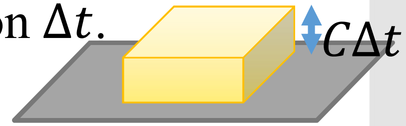
Light has a momentum?



$\vec{p} = k\hbar\hat{p} = h/\lambda\hat{p}$

Calculate the force that light strikes upon the wall – the pressure:

A volume of light strikes on the wall in a time duration Δt.



The total energy is $K = (AC\Delta t)(u_{EM})$

The momentum variation is $p_{total} = \frac{K}{C} = Au_{EM}\Delta t$

The pressure on the wall for a complete absorption of light is

$$P_{ressure} = \frac{p_{total}}{A\Delta t} = u_{EM} \rightarrow \boldsymbol{P_{ressure} = u_{EM} = \frac{I}{C} = \frac{P_{ower}}{AC}}$$

What about the force on the wall? $\boldsymbol{F = P_{ressure}A = \frac{P_{ower}}{C}}$

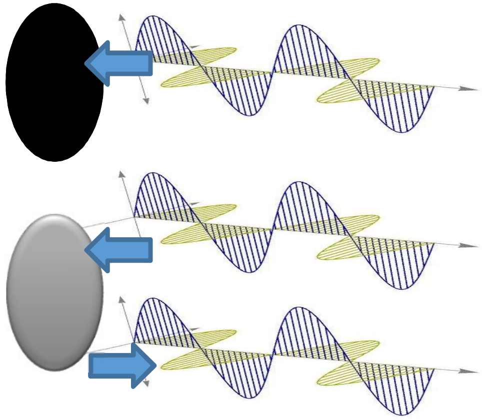
Mirror Reflection & Laser Tweezer

For a perfect absorber, the pressure is

$$P_{ressure} = \frac{I}{C}$$

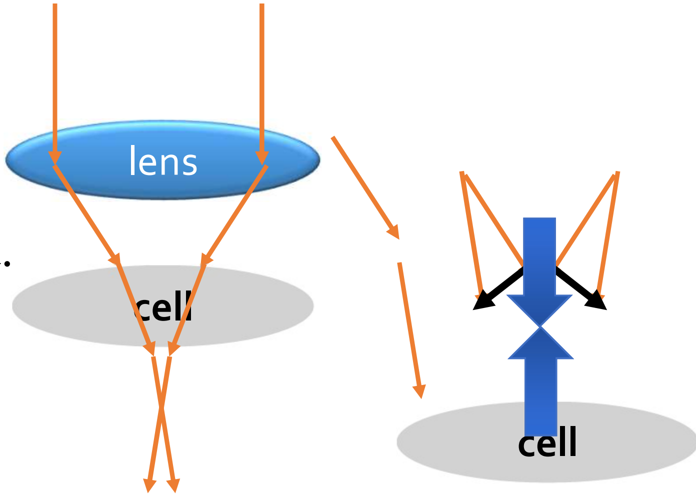
For a perfect reflector, the pressure is

$$P_{ressure} = 2\frac{I}{C}$$



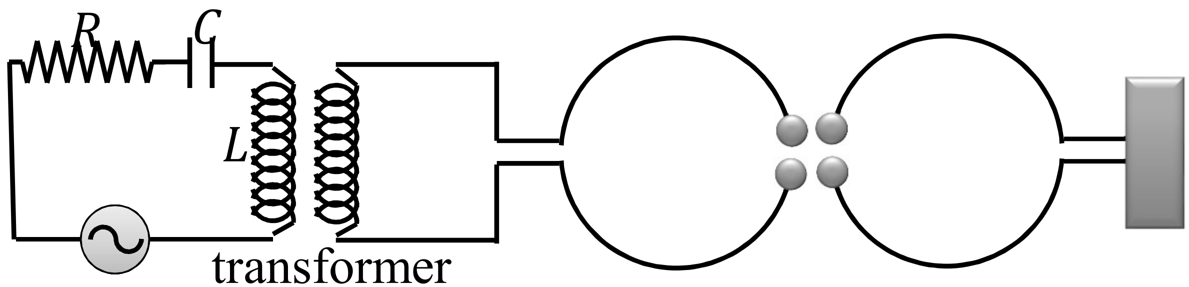
How does the laser tweezer work?

Use Newton's third law:
The cell pull the light downward.
Thus the light push the cell upward.



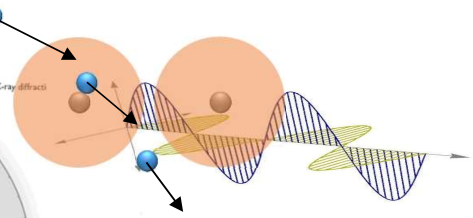
Generating EM Waves

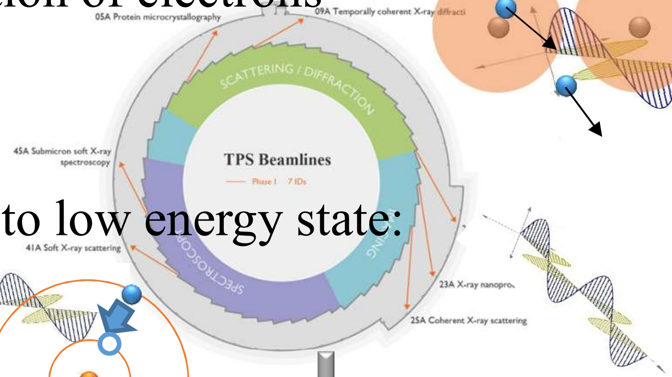
Use the LC oscillator:



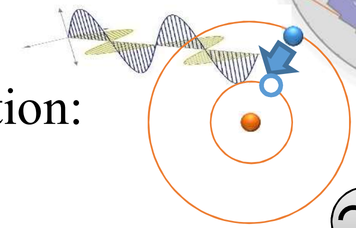
Accelerating or deaccelerating free charges:

- bremsstrahlung: deceleration of electrons by metals
- synchrotron radiation

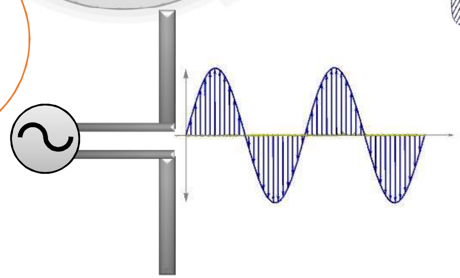




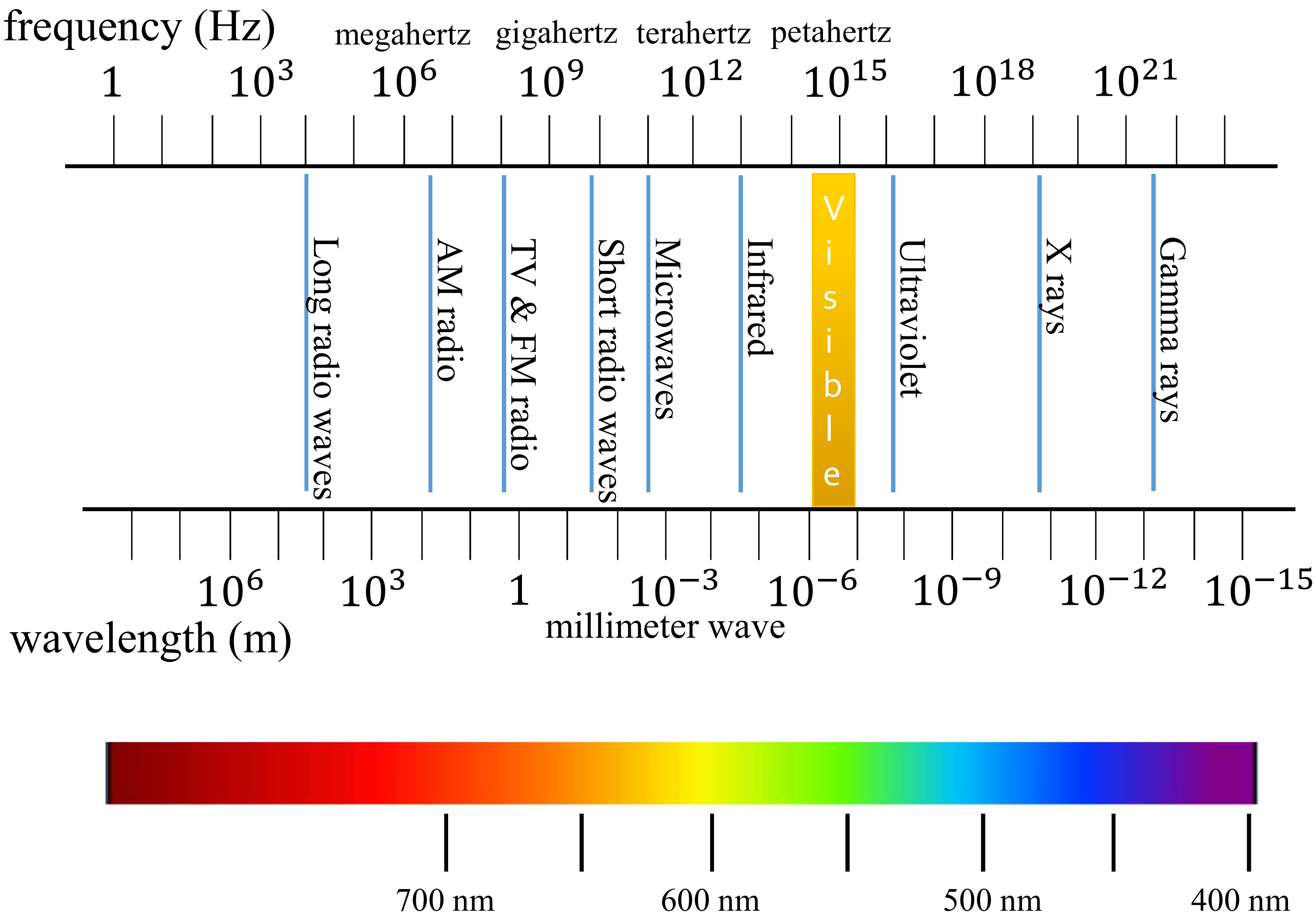
Transition from high energy to low energy state:



Electric dipole radiation:



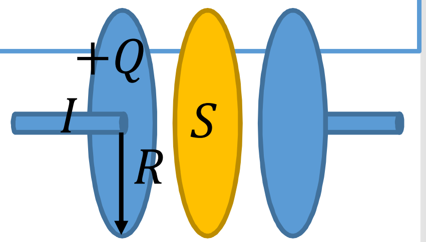
The Spectrum of EM Waves



Calculate The Displacement Current

Examples

A parallel plate capacitor has closely spaced circular plates of radius R. Charge is flowing onto the positive plate and off the negative plate at the rate I. Compute the displacement current through surface S passing between the plates by directly computing the rate of change of the flux of E through surface S.



$$\sigma = \frac{Q}{\pi R^2} \rightarrow E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\pi \varepsilon_0 R^2}$$

$$\Phi_E = E\pi R^2 = \frac{Q}{\varepsilon_0}$$

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{dQ}{dt} = I$$

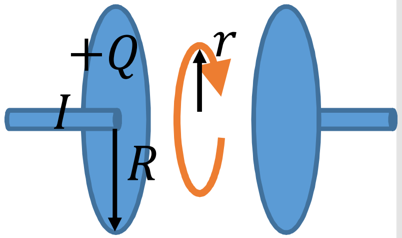
Ref:

Calculate The Magnetic Field

Examples

The circulate plates have a radius of R = 3.0 cm. Find the magnetic field strength B at a point between the plates a distance r = 2.0 cm from the axis of the plates when the current into the positive plate is 2.5 A.

$$\sigma = \frac{Q}{\pi R^2} \rightarrow E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\pi \varepsilon_0 R^2}$$



$$\Phi_E = E\pi r^2 = \frac{Q}{\varepsilon_0}\frac{r^2}{R^2}$$

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{r^2}{R^2}\frac{dQ}{dt} = \frac{2^2}{3^2}(2.5) = 1.11\ (\mathrm{A})$$

$$B(2\pi r) = \mu_0 \frac{r^2}{R^2} I \rightarrow B = \frac{\mu_0}{2\pi}\frac{rI}{R^2} = 4.93 \times 10^{-6}\ (\mathrm{T})$$

Ref:

Examples

The electric field of an electromagnetic wave is given by $\vec{E}(x,t) = E_0 \cos(kx - \omega t)\,\hat{k}$. (a) What is the direction of propagation of the wave? (b) What is the direction of the magnetic field in the $x = 0$ (yz) plane at time $t = 0$? (c) Find the magnetic field of the same wave. (d) Compute $\vec{E} \times \vec{B}$.

(a) propagation in the $\hat{x}$ direction

(b) the direction of magnetic field is in the $-\hat{j}$ direction since $\hat{k} \times (-\hat{j}) = \hat{\imath}$

(c) $\vec{B}(x,t) = -\frac{E_0}{c}\cos(kx - \omega t)\,\hat{j}$

(d) $\vec{E} \times \vec{B} = \frac{E_0^2}{c}\cos^2(kx - \omega t)\,\hat{\imath}$

Ref:

Examples

The electric field of an electromagnetic wave is given by $\vec{E}(x,t) = \hat{j}E_0\sin(kx-\omega t) + \hat{k}E_0\cos(kx-\omega t)$. (a) Find the magnetic field of the same wave. (b) Compute $\vec{E}\cdot\vec{B}$ and $\vec{E}\times\vec{B}$.

(a) From $\vec{E}(x,t)$, we know that the propagation is in the $\hat{\imath}$ direction.

The magnitude of B is E/C and the direction of B is in $\hat{\imath}\times\vec{E}$:

$$\hat{\imath}\times\vec{E} = \hat{k}E_0\sin(kx-\omega t) - \hat{j}E_0\cos(kx-\omega t)$$

$$\vec{B} = \hat{k}\frac{E_0}{C}\sin(kx-\omega t) - \hat{j}\frac{E_0}{C}\cos(kx-\omega t)$$

(b) Because $\vec{E}$ and $\vec{B}$ are perpendicular, $\vec{E}\cdot\vec{B} = 0$.

$$\vec{E}\times\vec{B} = \hat{\imath}\left(\frac{E_0^2}{C}\sin^2(kx-\omega t) + \frac{E_0^2}{C}\cos^2(kx-\omega t)\right) = \hat{\imath}\frac{E_0^2}{C}$$

Examples

A point source of electromagnetic radiation has an average power output of 800 W. Calculate the maximum values of the electric and magnetic fields in a vacuum at a point 3.5 m from the source.

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{800}{4\pi(3.5)^2} = 5.2\ \mathrm{W/m^2}$$

$$I = Cu_{EM} = C\frac{\varepsilon_0 E_0^2}{2} \rightarrow E_0 = \sqrt{2I/C\varepsilon_0} = 62.6\ \mathrm{V/m}$$

$$B_0 = \frac{E_0}{C} = 2.09 \times 10^{-7}\ \mathrm{T}(= \mathrm{V\,s\,m^{-2}})$$

Examples

The sun delivers 1,000 W/m^2 of energy to the Earth's surface. (a) Calculate the total power incident on a roof of dimension 10 m x 20 m.

$$P = IA = 1000 \times 10 \times 20 = 2 \times 10^5 \text{ W}$$

If a 5 mW pointer has a spot diameter of 2 mm. Please calculate the radiation pressure on a screen that reflects 70% of the light striking it.

$$I = 5 \times 10^{-3}/\pi\left(10^{-3}\right)^2 = 1.59 \times 10^3 \text{ W/m}^2$$

$$P_{ressure} = u_{EM} = I/C$$

For a screen reflecting 70% of light, the light pressure is

$$P_{ressure} = \frac{1.7I}{C} = 9.02 \times 10^{-6} \text{ N/m}^2$$

Examples

You get stuck in the space with a distance of s away from your spaceship. You are lucky and carrying a laser of power P with you. If the total mass is m, how long will you get back to the spaceship with the laser pointing directly away from the spaceship?

$$P_{ressure} = \frac{I}{C} = \frac{P}{AC} \rightarrow F = P_{ressure}A = \frac{P}{C}$$

$$F = \frac{P}{C} = ma \rightarrow a = \frac{P}{mC}$$

$$s = \frac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2smC}{P}}$$

Examples

A loop antenna consisting of a single 10-cm radius loop of wire is used to detect electromagnetic waves for which $E_{rms} = 0.15$ V/m. Find the rms emf induced in the loop if the wave frequency is (a) 600 kHz and (b) 60 MHz.

$$B_{rms} = \frac{E_{rms}}{C} = 5.0 \times 10^{-10}\ \mathrm{T}$$

$$B(x,t) = B_0 \sin(kx - \omega t) \rightarrow \Phi_B = AB_0 \sin(kx - \omega t)$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = \omega AB_0 sin(kx - \omega t) \rightarrow \varepsilon_{rms} = \omega AB_{rms}$$

(a) $\varepsilon_{rms} = 2\pi \times 6 \times 10^5\left(\pi(10^{-1})^2\right) \times \left(5.0 \times 10^{-10}\right) = 5.92 \times 10^{-5}\ \mathrm{V}$

(b) $\varepsilon_{rms} = 2\pi \times 6 \times 10^7\left(\pi(10^{-1})^2\right) \times \left(5.0 \times 10^{-10}\right) = 5.92 \times 10^{-3}\ \mathrm{V}$

Examples

(a) Please calculate the photon energy of a visible light with a wavelength of 400 nm. (b) Please calculate the photon energy of a x-ray with a wavelength of 0.1 nm.

(a) $E = hf = \frac{hC}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^{8}}{400\times10^{-9}} = 4.97 \times 10^{-19}$ J

$$E = \frac{4.97 \times 10^{-19}}{1.602 \times 10^{-19}} = 3.10 \text{ eV}$$

(b) $E = hf = \frac{hC}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^{8}}{0.1\times10^{-9}} = 1.99 \times 10^{-15}$ J

$$E = \frac{1.99 \times 10^{-1}}{1.602 \times 10^{-19}} = 12.4 \text{ keV}$$