Chapter 38 Relativity

Physics II – Part III

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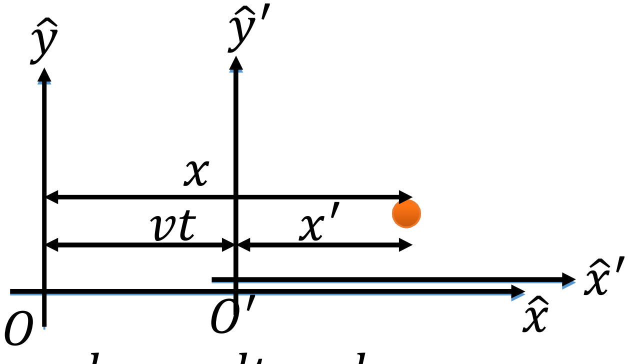
Galilean Relativity, Ether Wind

Galiean's Principles of Relativity:

The laws of mechanics must be the same in all inertial frames of reference.

$$x' = x - vt \qquad x = x' + vt'$$
$$y' = y \quad \Leftrightarrow \quad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = t \qquad t = t'$$



$$dx' = dx - vdt \; \& \; dt' = dt \rightarrow \frac{dx'}{dt'} = \frac{dx - vdt}{dt} = \frac{dx}{dt} - v$$

$$v'_x = v_x - v$$

With Galiean's principle, the speed of light can be higher than the ideal light speed C like $C + v$, where v is the relative speed between two frames of reference.

Existence of Ether or Not

Assume that electromagnetic waves propagate in a medium named ether. It is like that the plane acts on the air to fly while the air moves against the Earth ground.

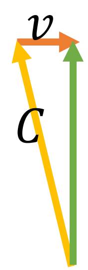
The motion of ether is called ether wind.

Light, moving downwind with a ether wind speed of v, gives a final speed of $C + v$

Light, moving upwind with a ether wind speed of v, gives a final speed of $C - v$

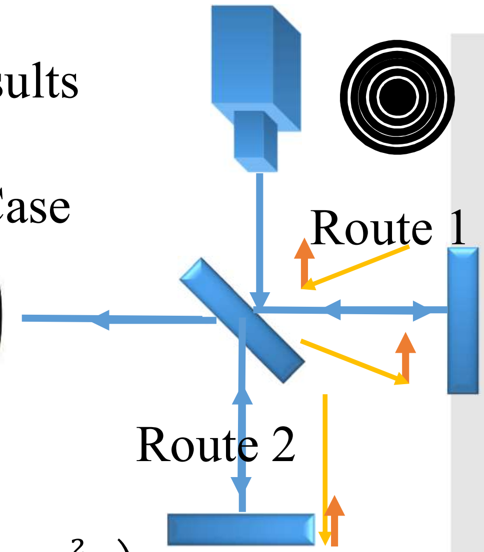
Light, moving perpendicular to the ether wind, gives a final speed of

$$\sqrt{C^2 - v^2}$$



Existence of Ether or Not – The Michelson-Morley Experiment

Michelson had some preliminary experimental results in 1881.
He collaborated with Morley when he moved to Case School of Applied Science, Cleveland, USA.
CaseWesternReserve University



The equipment was set up on a table floating on mercury for easy rotation.

Route 1: $\Delta t_1 = \frac{2L}{\sqrt{C^2 - v_{eth}^2}} = \frac{2L}{C}\left(1 - \frac{v_{eth}^2}{C^2}\right)^{-1/2} \cong \frac{2L}{C}\left(1 + \frac{v_{eth}^2}{2C^2}\right)$

Route 2: $\Delta t_2 = \frac{L}{C + v_{eth}} + \frac{L}{C - v_{eth}} = \frac{2CL}{C^2 - v_{eth}^2} = \frac{2L}{C}\left(1 - \frac{v_{eth}^2}{C^2}\right)^{-1} \cong \frac{2L}{C}\left(1 - \frac{v_{eth}^2}{C^2}\right)$

Time difference: $\Delta t_2 - \Delta t_1 = \frac{2L}{C}\frac{v_{eth}^2}{2C^2} = \frac{Lv_{eth}^2}{C^3}$

A rotation doubles the time difference.

The time difference gives a phase difference of $\frac{2Lv_{eth}^2}{C^3}\frac{2\pi}{T} = 2\pi\frac{2Lv_{eth}^2}{C^3}\frac{C}{\lambda}$

Einstein’s Principles of Relativity

Principles of Relativity

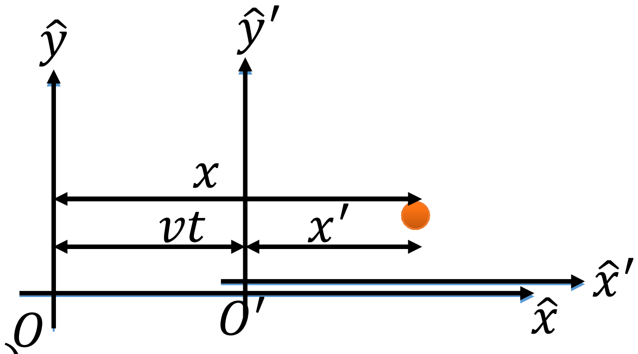
Galilean Space-Time Transformation

$x' = x_{PO'} = x - vt \qquad x' = C(x - vt)$

$y' = y$

$z' = z$

$t' = t \qquad t' = C(t - kvx)$



Two Principles to Derive The Einstein’s Relativity

1. The Principle of Relativity:

The laws of physics must be the same in all inertial frames of reference.

2. The Constancy of The Speed of Light:

The speed of light in vacuum has the same value in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Ref:

Consequences of Special Relativity

The speed of light c in O' is the same as that in O.

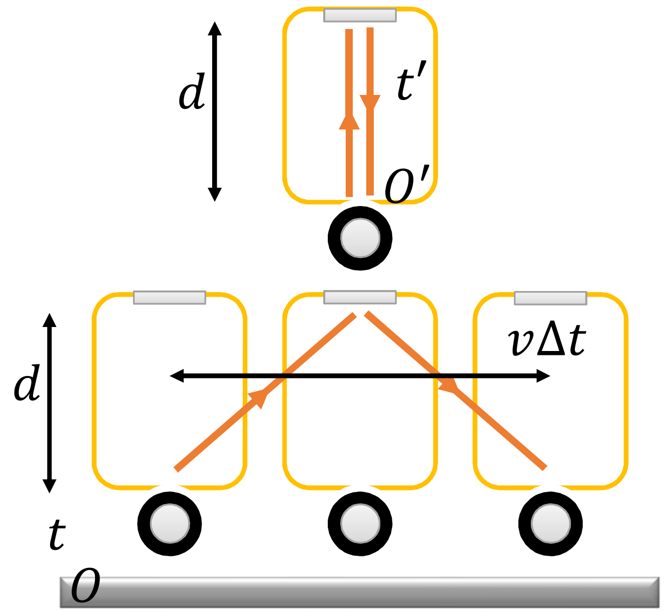
The time elapsed in O′

$$\Delta t' = \frac{2d}{C}$$

$$\frac{\Delta t}{2} = \frac{\sqrt{d^2 + \left(v\left(\frac{\Delta t}{2}\right)\right)^2}}{C}$$

$$\frac{C^2(\Delta t)^2}{4} = d^2 + \frac{v^2(\Delta t)^2}{4} \rightarrow (\Delta t)^2 = 4\frac{d^2}{C^2 - v^2}$$

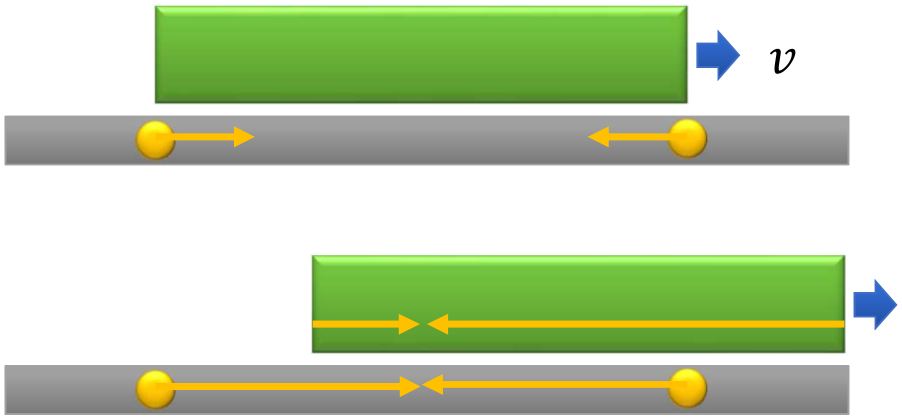
$$\Delta t = 2\frac{d}{\sqrt{C^2 - v^2}} = \frac{2d}{C}\frac{1}{\sqrt{1 - v^2/C^2}} = \Delta t' \gamma \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{C^2}}}$$



Ref:

Simultaneity

The two events of light emission, that are simultaneous in the frame at rest, are not simultaneous in the frame in motion.



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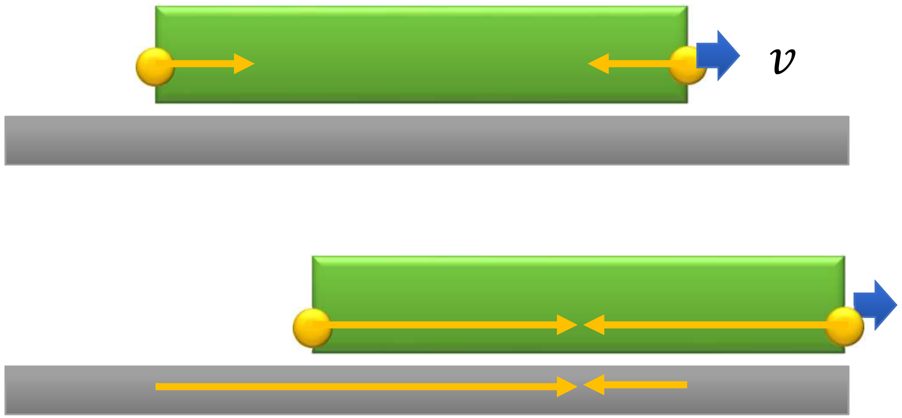


Time Dilation – Absolute Rest Frame

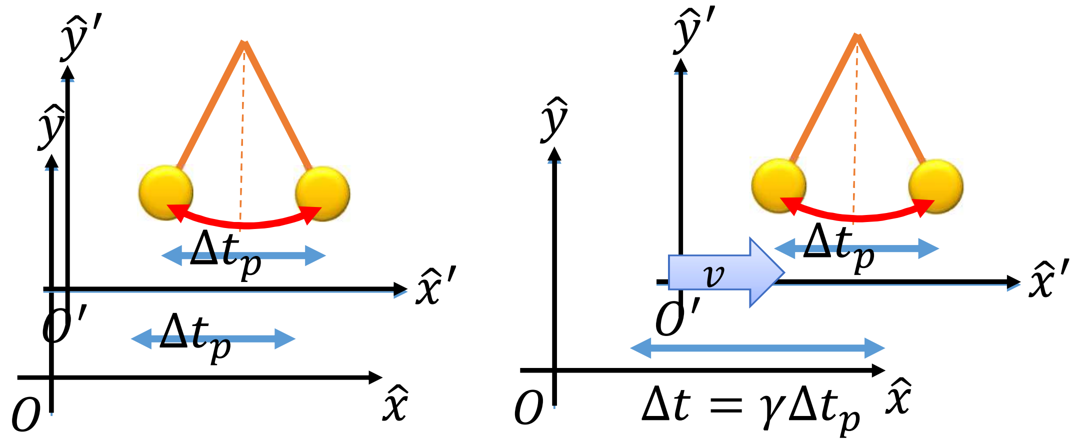
Consequences of Special Relativity

https://www.symmetrymagazine.org/article/may-2014/the-twin-paradox

- There must be an absolutely inertial frame at rest.
- Time measurement in System A: Δt_A, in system B: Δt_B
- If System A is at rest with respect to the absolutely inertial frame and System B is moving, the conversion of time in system B to A is $\Delta t_A = \gamma \Delta t_B$. The measured time in B is shorter.
- We see in the System A, the time is pushed longer due to a longer distance traveled by light.

Ref:

Consequences of Special Relativity

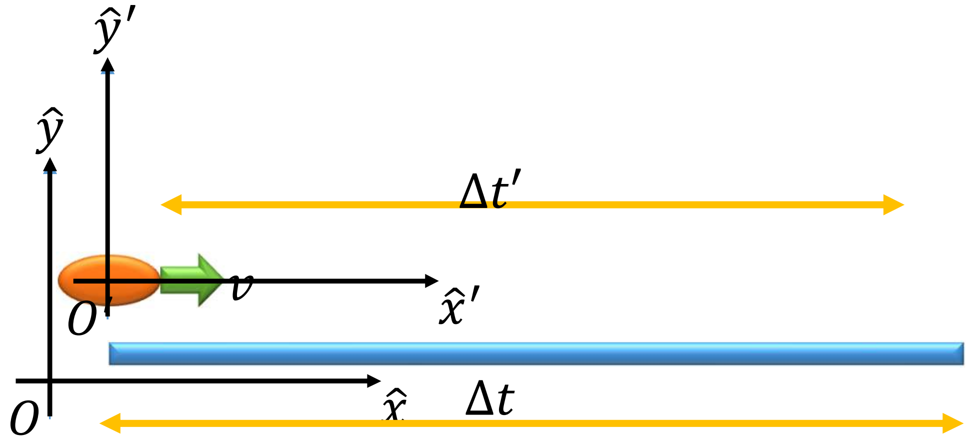


- If an event is measured at rest (shorter) in System A and measured at a constant moving speed in System B, the conversion of the event elapsed time measured in System A and B is $\Delta t_B = \gamma \Delta t_A$.
- Decay time of muons at rest is much shorter than that of muons moving at a speed close to C.

Ref:

Length Contraction

Consequences of Special Relativity



- The length measurement is an event!
- The length measurement are done in either a system at rest or another system at a high speed.
- The measurement is estimated by $v\Delta t$
- The measured time in the moving system $\Delta t'$ is shorter.
- The measured time in the system at rest Δt is longer.
- Consequently, you see the moving object is shorter ($C\Delta t'$) then it is at rest ($C\Delta t$). $\rightarrow L = \gamma L'$

Ref:

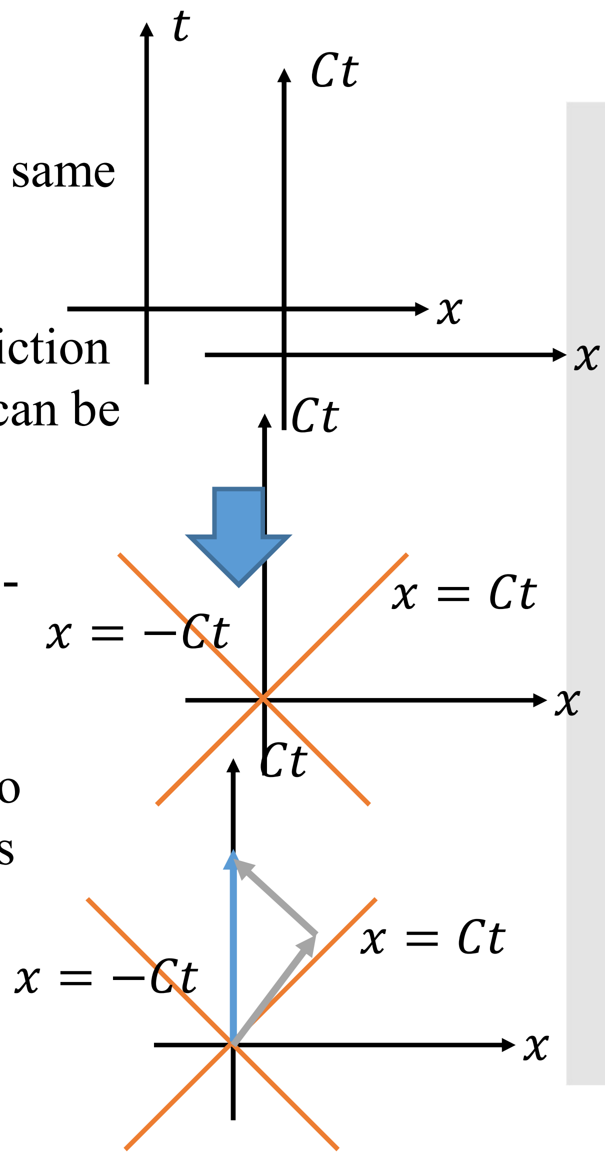
Space-Time Graphs

Time t can be converted to Ct, showing the same unit as a distance.

With the two distance coordinates, the restriction of the speed smaller than the light speed C can be drawn on it.

All physical phenomena occurs above the x-axis between the two lines of $x = \pm Ct$.

For example, the space-time graph is used to show the physical states of the twin brothers in the twin paradox story.



Transformation of Position and Time

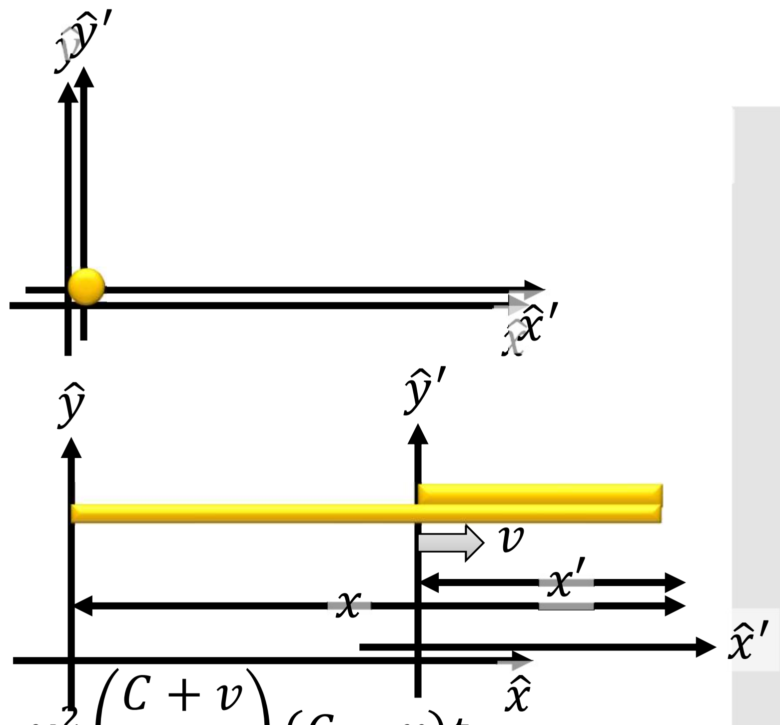
Lorentz Transformation Equation

Try to find a transformation:

$$x' = \gamma(x - vt)$$

$$x = \gamma(x' + vt')$$

Assume a light emits from
$x = x' = 0$ at $t = t' = 0$



$$Ct' = \gamma(Ct - vt)$$

$$Ct = \gamma(Ct' + vt') = \gamma\left(1 + \frac{v}{C}\right)Ct' = \gamma^2\left(\frac{C+v}{C}\right)(C-v)t$$

$$\gamma^2 = \frac{C^2}{C^2 - v^2} = \frac{1}{1-(v/C)^2} \rightarrow \gamma = \frac{1}{\sqrt{1-(v/C)^2}}$$

$$x' = \frac{x - vt}{\sqrt{1-(v/C)^2}} \;\&\; x = \frac{x' + vt'}{\sqrt{1-(v/C)^2}}$$

Ref:

Transformation of Position and Time

Lorentz Transformation Equation

Lorentz' spatial transformation:

$$x' = \gamma(x - vt)$$

$$x = \gamma(x' + vt')$$

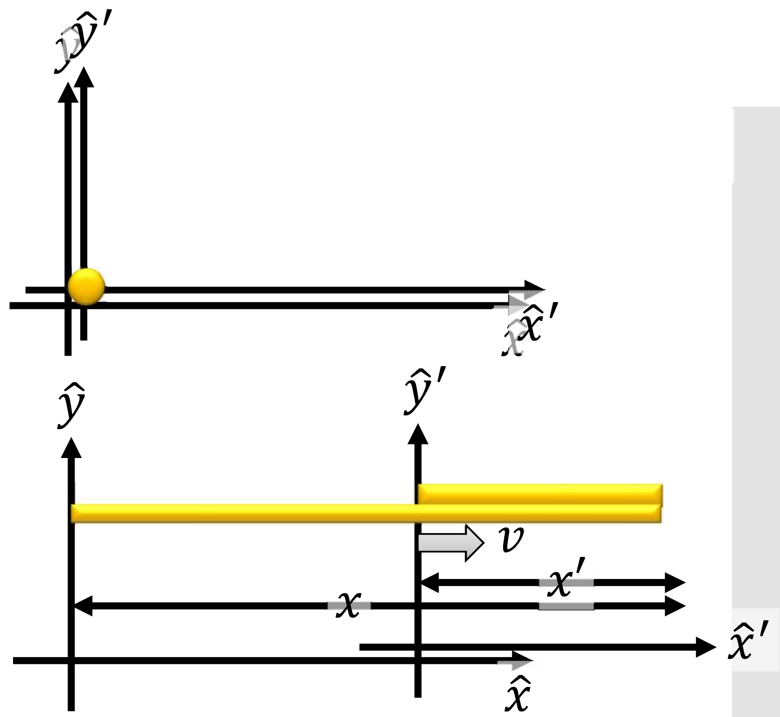
$$\frac{x'}{\gamma} = x - vt \rightarrow t = \frac{1}{v}\left(x - \frac{x'}{\gamma}\right)$$

$$t = \frac{1}{v}\left(\gamma x' + \gamma v t' - \frac{x'}{\gamma}\right)$$

$$t = \frac{1}{\gamma v}\left((\gamma^2 - 1)x' + \gamma^2 v t'\right)$$

$$t = \frac{1}{\gamma v}\left(\frac{(v/C)^2}{1-(v/C)^2}x' + \gamma^2 v t'\right) = \frac{1}{\gamma v}\left(\gamma^2 \frac{v^2}{C^2}x' + \gamma^2 v t'\right)$$

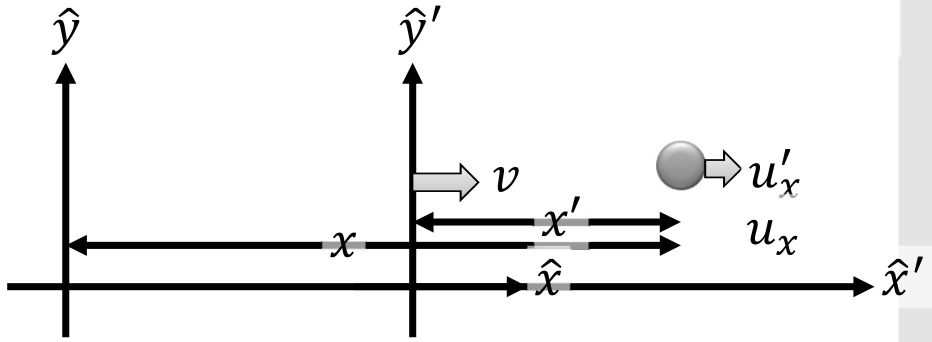
$$t = \gamma\left(t' + \frac{vx'}{C^2}\right) \quad t' = \gamma\left(t - \frac{vx}{C^2}\right)$$



Ref:

Velocity Transformation

The Lorentz Velocity Transformation Equation



Lorentz Transformation

$$x = \gamma(x' + vt') \Rightarrow dx = \gamma(dx' + vdt')$$

$$t = \gamma\left(t' + \frac{v}{C^2}x'\right) \Rightarrow dt = \gamma\left(dt' + \frac{v}{C^2}dx'\right)$$

$$\frac{dx}{dt} = u_x = \frac{dx' + vdt'}{dt' + \frac{v}{C^2}dx'} = \frac{u_x' + v}{1 + \frac{v}{C^2}u_x'}$$

$$u_x' = \frac{u_x - v}{1 - \frac{v}{C^2}u_x}$$

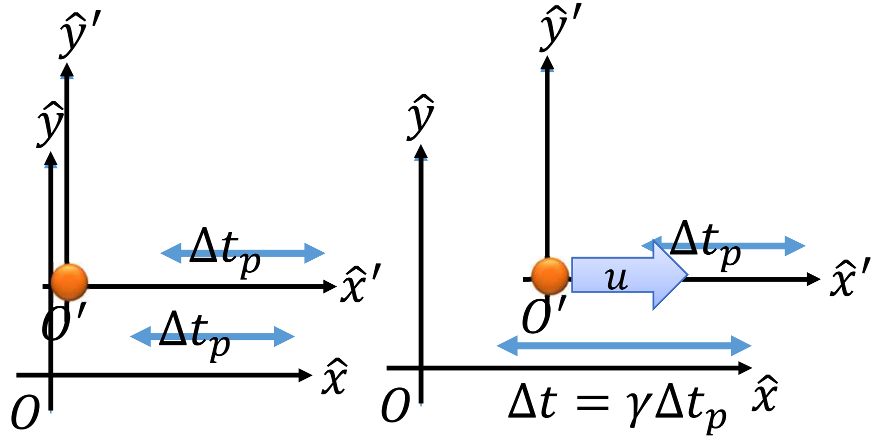
Ref:

Momentum & Energy

Non-relativistic momentum:

$p = mu$

In the particle system (O' system), the velocity $u' = \Delta x/\Delta t'$.



The time measured in the observer O system at rest is Δt but the particle experiences a time interval of $\Delta t/\gamma$ due to the relativity.

$$u' = \frac{\Delta x}{\Delta t'} = \frac{\Delta x}{\Delta t/\gamma} = \gamma u$$

The momentum of the particle is

$$p = mu' = m\gamma u = \gamma m u = \frac{mu}{\sqrt{1-(u/C)^2}}$$

Ref:

Relativistic Linear Momentum & Energy

Momentum & Energy

Derive the relativistic energy from the relativistic linear momentum.

$$p = \frac{mu}{\sqrt{1 - u^2/C^2}}$$

The laws of physics do not change. $F = \frac{dp}{dt}$

$$K = \int F dx = \int \frac{dp}{dt} dx = \int \frac{dp}{dt} u dt = \int_0^{u'} u dp$$

$$K = \left[\frac{mu^2}{\sqrt{1 - \frac{u^2}{C^2}}}\right]_0^{u'} - \int_0^{u'} \frac{mu}{\sqrt{1 - \frac{u^2}{C^2}}} du = \frac{mu'^2}{\sqrt{1 - \frac{u'^2}{C^2}}} + mC^2 \left[\sqrt{1 - \frac{u^2}{C^2}}\right]_0^{u'}$$

$$K = \frac{1}{\sqrt{1 - \frac{u'^2}{C^2}}}\left(mu'^2 - mC^2\sqrt{1 - \frac{u'^2}{C^2}} + mC^2 - mu'^2\right) = \frac{mC^2}{\sqrt{1 - \frac{u'^2}{C^2}}} - mC^2$$

Ref:

Mass & Energy

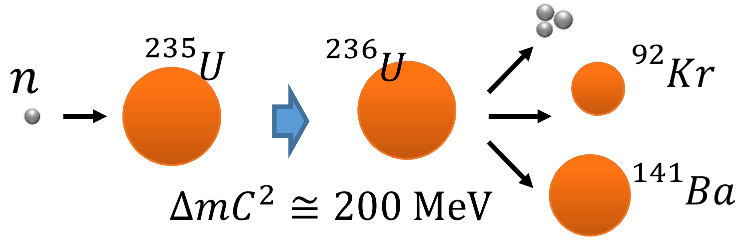
The total energy of a particle moving at an extremely high speed u is $\gamma mC^2 = mC^2/\sqrt{1-u^2/C^2}$.

The rest energy of a particle moving at zero speed is mC^2.

The kinetic energy of the high-speed moving particle is $\gamma mC^2 - mC^2$.

Here comes a new idea of mass-energy relationship of $\Delta E = \Delta mC^2$.

You can find some nuclear fission reaction that transfers the loss of mass to energy.



You can also find some other nuclear fusion reaction. For example, two deuterium atoms can combine to form a helium atom. The loss of mass is $\Delta m \cong 4.25 \times 10^{-29}$ kg and the generated energy is 23.9 MeV.

Derive The Energy Momentum Relation

Momentum & Energy

The relativistic linear momentum is $p = \dfrac{mu}{\sqrt{1 - u^2/C^2}}$

The total energy and the kinetic energy of the particle are

$$E = \frac{mC^2}{\sqrt{1 - u^2/C^2}} \; \& \; K = E - mC^2$$

Now remove the velocity u dependence in the momentum and energy expression.

$$\frac{p}{E} = \frac{u}{C^2} \rightarrow u = \frac{pC^2}{E}$$

$$E^2\left(1 - \frac{u^2}{C^2}\right) = m^2C^4 \rightarrow E^2\left(1 - \frac{p^2C^2}{E^2}\right) = m^2C^4$$

$$E^2 - p^2C^2 = m^2C^4 \rightarrow E^2 = p^2C^2 + m^2C^4$$

For a photon, $m = 0 \rightarrow E^2 = C^2p^2 \rightarrow E = Cp$

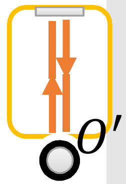
Ref:

General Theory of Relativity

Newton's law of gravitation is of the concept of instantaneous reaction that violates the law in the special relativity requiring the propagation no higher than the speed of light.

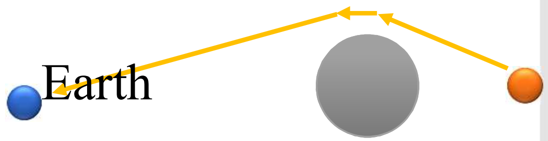
Einstein postulates the so-called principle of equivalence – the general physical law retains the same form.

Thus a system in a homogeneous gravitational field is completely equivalent to a uniformly accelerated reference system.



The next step relies on the equations for the motion in accelerated frames.

The most famous example is the bending of light by a massive object.



What is changed after considering the general theory of relativity?

- A clock runs slower due to the presence of gravity.
- The frequency of radiation emitted by atoms is redshifted to a lower frequency due to the gravity.

Time Dilation – Time of Event

Examples

The period of a pendulum is measured to be 3.00 s in the reference frame of the pendulum. What is the period when measured by an observer moving at a speed of 0.960c relative to the pendulum?

The interval measured at rest is 3.00 s.

$$\Delta t_p = 3.00$$

$$\Delta t = \gamma \Delta t_p = \frac{3}{\sqrt{1 - 0.960^2}} = 10.7$$

Ref:

Length Contraction

Examples

An astronaut takes a trip to Sirius, which is located a distance of 8 lightyears from the Earth. The astronaut measures the time of the one-way journey to be 6 years. If the spaceship moves at a constant speed of 0.8C, how can the 8-lightyear distance be reconciled with the 6-year trip time measured by the astronaut?

The 8 lightyears distance is measured at rest.

It will be $8 \times \sqrt{1 - \frac{(0.8C)^2}{C^2}} = 4.8$ lightyears, measured at a constant speed 0.8c.

The trip time is $\frac{4.8}{0.8} = 6$ years.

Examples

Use the Lorentz transformation equation to solve the following problem. Imagine two events that are simultaneous and separated in space such that $\Delta t' = 0$ and $\Delta x'$ according to an observer O' who is moving at speed v with respect to O. What is the simultaneity condition for an observer O.

Lorentz transformation gives $t = \gamma\left(t' + \frac{v}{C^2}x'\right)$.

In O', the two events occurs at time t_1', t_2' at places x_1', x_2', where $\Delta t' = t_1' - t_2' = 0$ and $\Delta x' = x_1' - x_2' \neq 0$.

For observers in O,

$$t_1 = \gamma\left(t_1' + \frac{v}{C^2}x_1'\right) \;\&\; t_2 = \gamma\left(t_2' + \frac{v}{C^2}x_2'\right)$$

$$\Delta t = t_1 - t_2 = \gamma\frac{v}{C^2}\Delta x' \neq 0$$

the two events are not simultaneous.

Examples

Use the Lorentz transformation equation to solve the following problem. A clock is at rest in O' while it is moving with a speed v in O. Check that the time measured in O' is shorter than that measured in O.

Lorentz transformation gives $t = \gamma\left(t' + \frac{v}{C^2}x'\right)$.

The clock is at rest in O' so we can think it as one event. We want to measure the time of the event in O' and compare it with that measured in O.

$x_1' = x_2' \rightarrow \Delta x' = 0$, the event goes from t_1' to t_2' and the elapsed time is $\Delta t' = t_2' - t_1'$

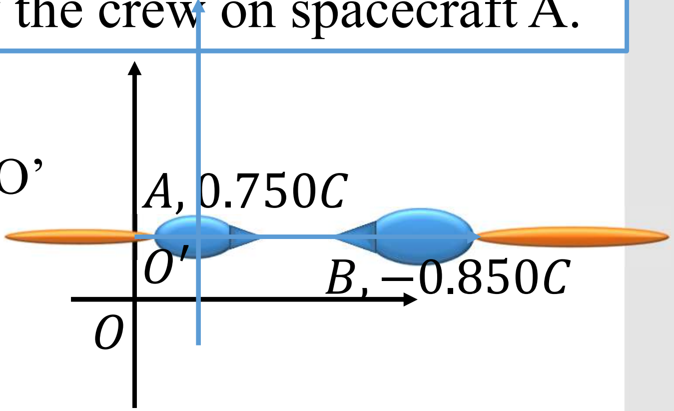
The same event elapsed time in O is $\Delta t = \gamma\left(\Delta t' + v\Delta x'/C^2\right) = \gamma\Delta t'$.

Examples

An observer on the Earth measures the speed of spacecraft A to be $0.750C$ and the speed of spacecraft B to be $-0.850C$. Find the velocity of spacecraft B as observed by the crew on spacecraft A.

Let O' be the reference frame of the spacecraft A. The relative speed between O' and O is $0.750C$.

Let u_x be the speed of the spacecraft B observed in O.



$$u'_x = \frac{u_x - v}{1 - \frac{v}{C^2} u_x} = \frac{-0.850C - 0.750C}{1 - \frac{0.750C}{C^2}(-0.850C)} = -0.977C$$

Examples

The mass of a proton is 1.6726×10^{-27} kg. (a) Find the rest energy of a proton in the unit of eV. (b) If the total energy of a proton is three times of its rest energy. What is the speed of the proton? (c) Determine the kinetic energy of the proton in the unit of eV. (d) What is the proton's momentum?

(a) $m_p = 1.6726 \times 10^{-27}$ kg, $E = m_p C^2 = 1.5053 \times 10^{-10}$ J $= 9.3967 \times 10^8$ eV

(b) $E = 3m_p C^2 \rightarrow 3 = \frac{1}{\sqrt{1-\frac{u^2}{C^2}}} \rightarrow \frac{u^2}{C^2} = 1 - \frac{1}{9} \rightarrow u = 2.8284 \times 10^8$ m/s

(c) $K = E - m_p C^2 = 2m_p C^2 = 2 \times 9.3967 \times 10^8 = 1.8793 \times 10^9$ eV

(d) $p = \gamma m u = 3 \times 1.6726 \times 10^{-2} \quad \times 2.8284 \times 10^8 = 1.4192 \times 10^{-18}$ kg m/s

Examples

The ^{216}Po nucleus is unstable and exhibits radioactivity. It decays to ^{212}Pb by emitting an alpha particle (^{4}He). The relevant mass are $m(216\text{Po}) = 216.001\,915\ u$, $m(4\text{He}) = 4.002\,603\ u$, and $m(212\text{Pb}) = 211.991\,898\ u$. (a) Find the mass change. (b) Find the energy from the fission reaction.

(a) u is the atomic unit, $u = 1.660539 \times 10^{-27}$ kg

$$216.001915u - 4.002603u - 211.991898u = 0.007414u$$

$$\Delta m = 0.007414u = 1.23112 \times 10^{-29}\ \text{kg}$$

(b) $1.2311 \times 10^{-29} \times \left(3 \times 10^{8}\right)^{2} = 1.1080 \times 10^{-1}\ \ \text{J}$