Chapter 39 Quantum Physics

Physics II – Part III
Wen-Bin Jian
Department of Electrophysics, National Chiao Tung University

Newtonian's statistics:

Basic Concept - Quantum

Newtonian's mechanics: $F = ma \rightarrow \frac{F}{m} = a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Solve the differential equation.

$$v(t) = \int_{t_0}^{t} a(t')dt' \qquad x(t) = \int_{t_0}^{t} v(t')dt'$$

Boltzmann's distribution: for a particle of energy $E = mv^2/2$,

the existence probability of the particle is $P(E) \propto \exp(-E/k_B T)$.

Let $P(E) = c\exp(-E/k_B T)$, the total probability is $\int_0^\infty c\exp(-E/k_B T)\,dE$. The classical statistics is introduced in the kinetic energy theory in Chapter 20.

Quantum statistics:

Basic Concept - Quantum

The frequency of light from a black body can be continuous.

With **a fixed light frequency f**, the probability according to the quantum statistics is detrived:

If there are no photons existed, the probability is $P(E) \propto e^0 = 1$.

For a single photon with energy $E = hf$, the existence probability is $P(E) \propto \exp(-hf/k_B T)$.

For a two-photon case, the probability is $P(E) \propto \exp(-2hf/k_B T)$.

For a three-photon case, the probability is $P(E) \propto \exp(-3hf/k_B T)$.

The total probability of the existence of all possible numbers of photons at a fixed frequency f is $P_T(E) = \sum_{n=0}^{\infty} \exp\left(-\frac{nhf}{k_B T}\right) = \frac{1}{1-\exp(-hf/k_B T)}$

Generating a Number of Photons

Black Body Radiation

For a light of a given frequency f, it's possible to give $E = 0, hf, 2hf, 3hf, \ldots$ and to generate zero, one, two, three, … photons.

The Boltzmann distribution gives the probability proportional to $P_0 = 1, P_1 = e^{-\frac{hf}{kT}}, P_2 = e^{-\frac{2hf}{kT}}, P_3 = e^{-\frac{3hf}{kT}}, \ldots.$

The average energy of the light with frequency f is

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nhf\, e^{-n\frac{hf}{kT}}}{\sum_{n=0}^{\infty} e^{-n\frac{hf}{kT}}}$$

Here $\sum_{n=0}^{\infty} e^{-n\frac{hf}{kT}}$ is the total probability of all events.

$$\sum_{n=0}^{\infty} e^{-n\frac{hf}{kT}} = \frac{1}{1 - e^{\frac{hf}{kT}}}$$

Ref:

Black Body Radiation

Generating a Number of Photons

$$\langle E\rangle = \frac{\sum_{n=0}^{\infty} nhf\; e^{-n\frac{hf}{kT}}}{\sum_{n=0}^{\infty} e^{-n\frac{hf}{kT}}}$$

Let $A = \frac{1}{kT}, \sum_{n=0}^{\infty} e^{-nAhf} = \frac{1}{1-e^{-Ahf}}$

$$-\frac{d}{dA}\left(\sum_{n=0}^{\infty} e^{-nA}\right) = \sum_{n=0}^{\infty} (nhf)e^{-nAh} = \left(1-e^{-Ahf}\right)^{-2} hf e^{-Ahf}$$

$$\langle E\rangle = \left(1-e^{-Ah}\right)^{-1} hf e^{-Ahf} = \frac{hf}{e^{Ahf}-1} = \frac{hf}{e^{\frac{hf}{kT}}-1}$$

If $kT \gg hf$, $e^{\frac{hf}{kT}} \cong 1 + \frac{hf}{kT}$ & $\langle E\rangle = \frac{hf}{e^{\frac{hf}{kT}}-1} \cong \frac{hf}{hf/kT} = kT$

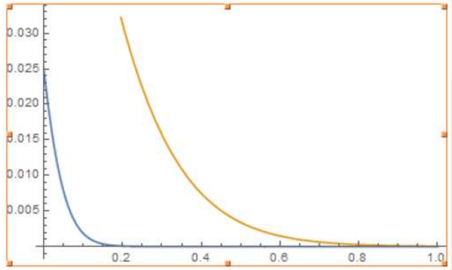
Ref:

Basic Concept – Momentum Space

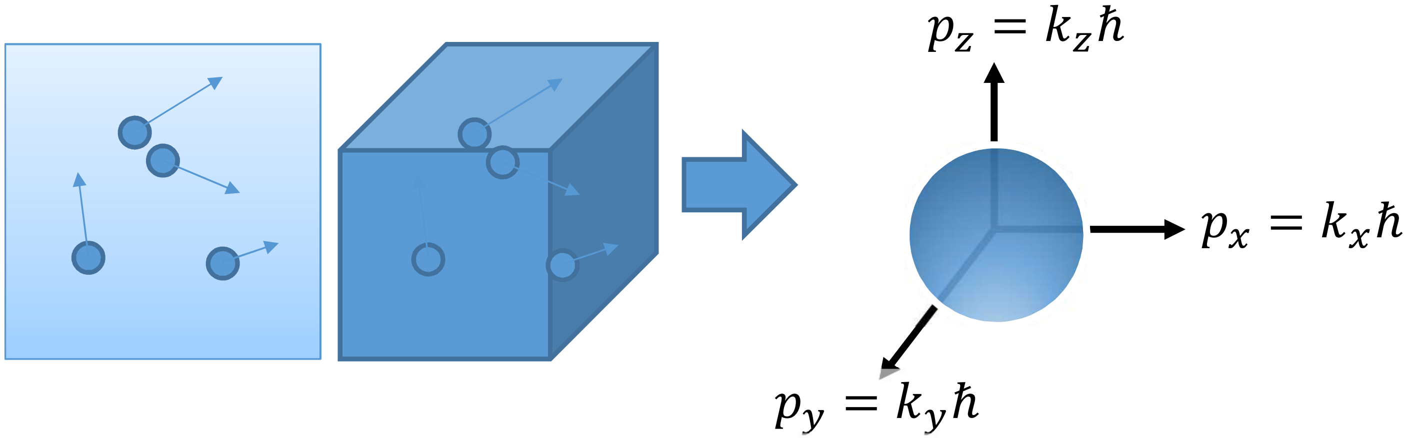
How can we estimate the number of particles having the same energy?

The classical limit occurs at $hf \ll k_B T$.

$$\langle E \rangle = \frac{hf}{\exp(hf/k_B T) - 1} \cong \frac{hf}{1 + \frac{hf}{k_B T} - 1} = k_B T$$



Real space versus momentum (velocity) space:



Electromagnetic waves are confined in a cube with a length of L. The quantization condition gives $kL = n\pi$.

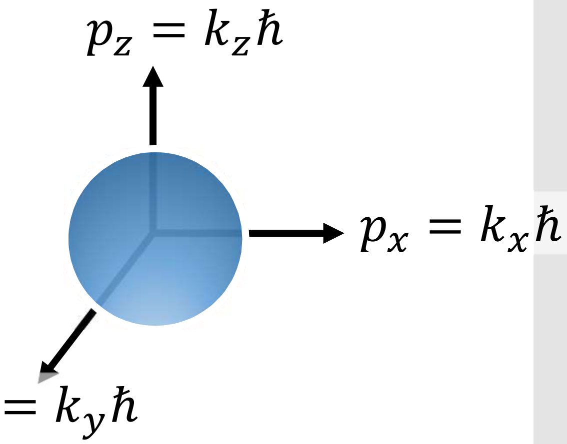
Basic Concept - Quantum

How can we estimate the number of particles having the same energy?

The condition of $kL = n\pi$ implies: $k > 0$ & $n = k\frac{L}{\pi} \rightarrow \Delta n = \Delta k\left(\frac{L}{\pi}\right)$.

The number of particles of the same energy is:

$4\pi k^2 dk \times \frac{1}{8} \times \left(\frac{L}{\pi}\right)^3 \times 2$ (2 polarizations for EM waves)



For a specified frequency of light, the energy per unit volume is

$$\frac{k^2}{\pi^2}dk \times \langle E\rangle = \frac{hf}{\exp(hf/k_BT) - 1}\frac{k^2}{\pi^2}dk, k = 2\pi/\lambda$$

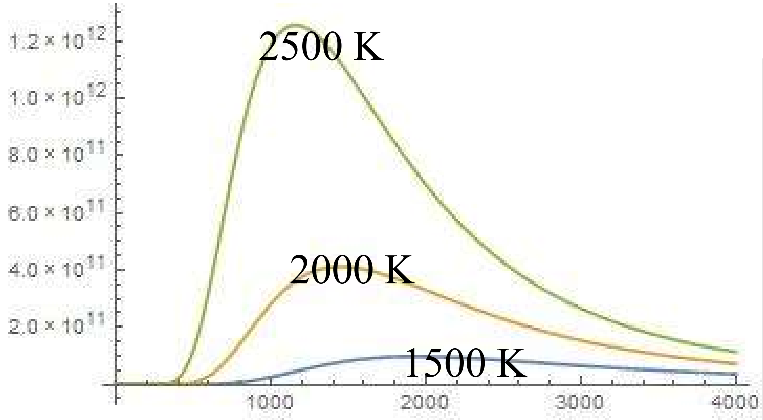
$$\frac{k^2}{\pi^2}\langle E\rangle dk = \frac{8\pi}{\lambda^4}\frac{hf}{\exp(hf/k_BT) - 1}d\lambda = \frac{8\pi}{\lambda^5}\frac{hc}{\exp(hc/\lambda k_BT) - 1}d\lambda$$

$$I(\lambda, T) = \frac{c}{4}\frac{8\pi}{\lambda^5}\frac{hc}{\exp(hc/\lambda k_BT) - 1} = \frac{2\pi hc^2}{\lambda^5(\exp(hc/\lambda k_BT) - 1)}$$

Temperature Dependence of The Intensity Distribution

The Peak of The Distribution

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5(\exp(hc/\lambda k_B T) - 1)}$$



$$\frac{dI}{d\lambda} = 0 \rightarrow \frac{d}{d\lambda}\left(\lambda^{-5}\left(\exp\left(\frac{A}{\lambda}\right) - 1\right)^{-1}\right) = 0, A = \frac{hc}{k_B T}$$

$$\rightarrow \left(\frac{A}{\lambda_{max}} - 5\right)e^{\frac{A}{\lambda_{max}}} + 5 = 0 \rightarrow \frac{A}{\lambda_{max}} = 4.96511 \rightarrow \lambda_{max} = \frac{hc}{4.96511 k_B T}$$

$$\rightarrow \lambda_{max} T = \frac{hc}{4.96511 k_B} = 2.9 \times 10^{-3} (\text{m K})$$

$$\lambda_{max} T = 2.898 \times 10^{-3}\ \text{m K}$$

Stephan's Law

$$I(\lambda, T) = \frac{1}{A}\frac{dP}{d\lambda} = \frac{2\pi hc^2}{\lambda^5(\exp(hc/\lambda k_B T) - 1)}$$

$$P = 2\pi hc^2 A \int_0^\infty \frac{d\lambda}{\lambda^5(\exp(hc/\lambda k_B T) - 1)}$$

$$\text{Let } x = \frac{hc}{\lambda k_B T}, \lambda = \frac{hc}{x k_B T} \;\&\; d\lambda = -\frac{hc}{k_B T}\frac{1}{x^2}dx$$

$$P = 2\pi hc^2 A \int_\infty^0 \left(\frac{x k_B T}{hc}\right)^5 \frac{-\frac{hc}{k_B T}\frac{1}{x^2}}{(\exp(x) - 1)} dx$$

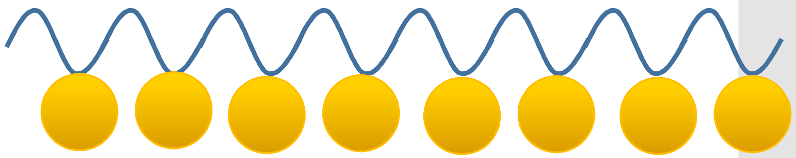
$$P = 2\pi hc^2 A \left(\frac{k_B T}{hc}\right)^4 \int_0^\infty \frac{x^3}{(\exp(x) - 1)} dx$$

$$P = 2\pi hc^2 A \left(\frac{k_B T}{hc}\right)^4 \frac{\pi^4}{15} = \frac{2\pi^5 k_B^4 A}{15 h^3 c^2} T^4 = \sigma A e T^4, e = 1$$

The Photoelectric Effect

Energy Quantization of Light - Photon

Light is not just a plane wave of continuous energy distribution.

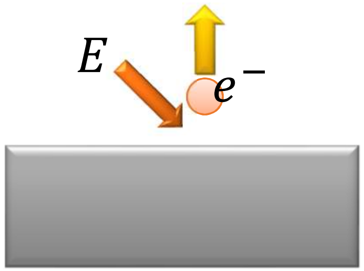


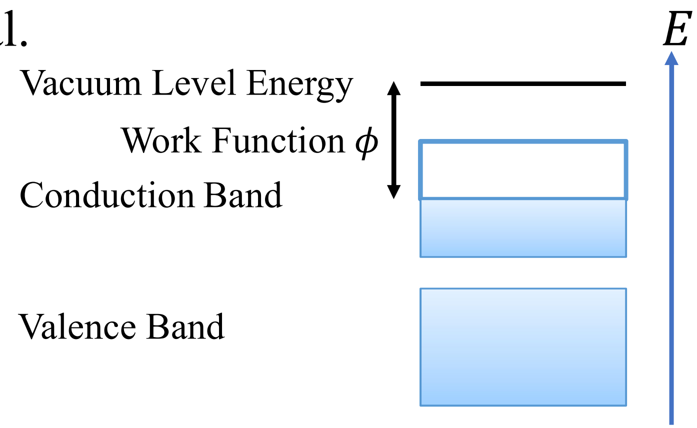
$$E = Vu_{EM} = V \times \frac{\varepsilon_0 E_0^2}{2}$$

Light is quantized as many photons.

$E = nhf$ n is number of photons, f is the frequency of light

The work function of metal is the required energy to move one electron away from the bulk metal.





Ref:

The Photoelectric Effect

Energy Quantization of Light - Photon

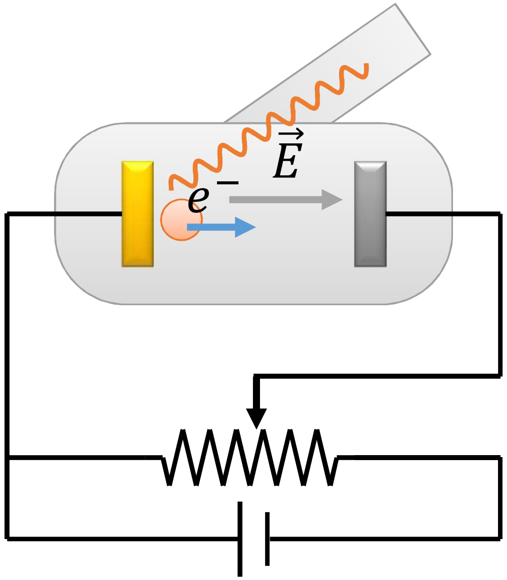
Light is applied to move electrons away from the metal bulk.

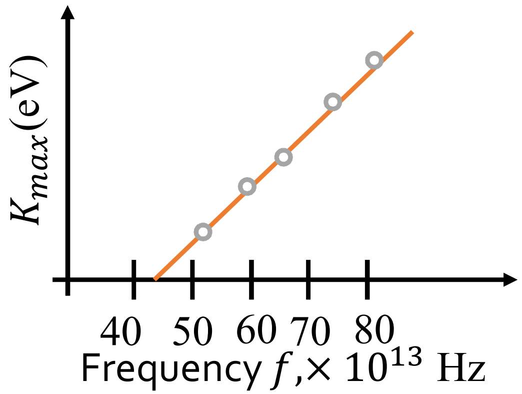
Energy of light E_{light} must be higher than the work function ϕ. $E_{light} > \phi$

If $E_{light} = Vu_{EM}$, we can increase the intensity or power of light to generate electrons.

The results show that the energy of light is only dependent on the frequency. It gives us another energy concept of $E = nhf$.

In addition, it indicates one photon for the excitation of one electron to move away from the metal. $E = hf, K = E - \phi = hf - \phi$

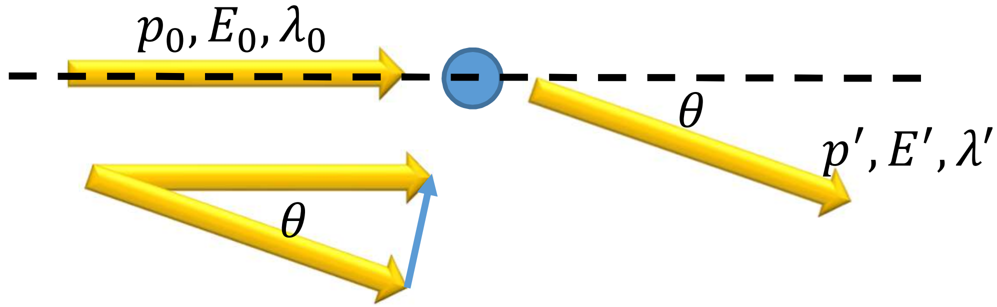




Ref:

Light As A Particle

After scattering → lower energy → longer wavelength



momentum conservation: $p_e^2 = p_0^2 + p'^2 - 2p_0 p' \cos\theta$

energy conservation: $cp_0 + m_e c^2 = cp' + \sqrt{m_e^2 c^4 + p_e^2 c^2}$

Ref:

The Compton Effect

Light As A Particle

$$p_e^2 = p_0^2 + p'^2 - 2p_0p'\cos\theta \qquad cp_0 + m_ec^2 = cp' + \sqrt{m_e^2c^4 + p_e^2c^2}$$

$$m_e^2c^4 + p_e^2c^2 = c^2p_0^2 + m_e^2c^4 + c^2p'^2 + 2p_0m_ec^3 - 2c^2p_0p' - 2m_ec^3p'$$

$$c^2p_0^2 + c^2p'^2 - 2c^2p_0p'\cos\theta = c^2p_0^2 + c^2p'^2 + 2p_0m_ec^3 - 2c^2p_0p' - 2m_ec^3p'$$

$$-2c^2p_0p'\cos\theta = 2p_0m_ec^3 - 2c^2p_0p' - 2m_ec^3p'$$

$$-p_0p'\cos\theta = p_0m_ec - p_0p' - m_ecp'$$

$$-\cos\theta = \frac{m_ec}{p'} - 1 - \frac{m_ec}{p_0} \qquad 1 - \cos\theta = \frac{m_ec}{p'} - \frac{m_ec}{p_0}$$

$$1 - \cos\theta = \frac{m_ec}{h/\lambda'} - \frac{m_ec}{h/\lambda_0} \qquad \lambda' - \lambda_0 = \frac{h}{m_ec}(1 - \cos\theta)$$

Ref:

Photons and Electromagnetic Waves

Wave-Particle Dual Natures

The photoelectric effect and the Compton effect show the particle nature of light.

The wave nature of light gives us the wavelength and the frequency of the light wave:

$$\vec{E}(x,t) = E_0 \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)\hat{j} \;\&\; \vec{B}(x,t) = \frac{E_0}{c}\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)\hat{k}$$

The relativistic energy-momentum relation is $E^2 = c^2p^2 + m^2c^4$.

The mass of light is zero thus $E = cp$.

Start from the quantized energy of light quanta $E = hf$ and the linear energy-momentum dispersion relation $E = cp$ we obtain the momentum-wavelength relation

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{hf}{f\lambda} = \frac{h}{\lambda}$$

Ref:

The de Broglie's Matter Wave

Wave-Particle Dual Natures

We learn from light about the dual nature and use it to express the wave nature of a particle. If the particle's energy E and linear momentum p are given, we have the wave properties:

$$f = \frac{E}{h} \ \& \ \lambda = \frac{h}{p} \ (k = \frac{2\pi}{\lambda} \rightarrow p = k\frac{h}{2\pi} = k\hbar)$$

Note that, different from light, the particle gives an energy-momentum relation of $E = \frac{p^2}{2m}$.

The wave function is $f(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - 2\pi f t + \phi\right)$, including an angle ϕ named phase.

$$f(x,t) = A\sin\left(\frac{2\pi p}{h}x - \frac{2\pi E t}{h} + \phi\right)$$

The particle velocity is p/m, the wave velocity is $v_p = f\lambda = \omega/k$, and the group velocity is $v_g = d\omega/dk$.

Ref:

Wave-Particle Dual Natures

Parameters of the wave nature: f and λ (frequency & wavelength)

After we know f and λ, we have the wave function

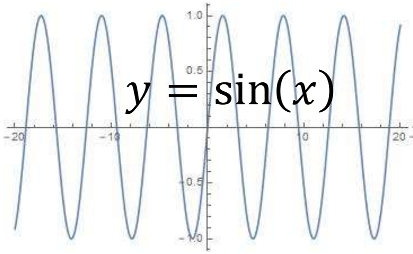
$$\psi(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - 2\pi f t + \phi\right) = A\sin(kx - \omega t + \phi)$$

Parameters of the particle nature: E and p (energy & momentum)

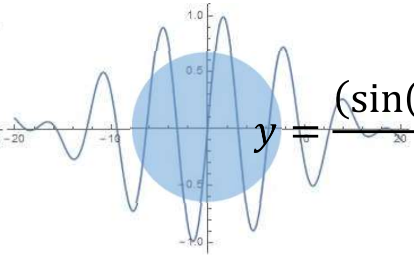
	f	λ
E	$E = hf$	
p		$p = \frac{h}{\lambda} = k\hbar$

	$E - p$ relation
particles	$E = p^2/2m$
electromagnetic wave (photons)	$E = cp$

Phase Velocity & Group Velocity



$y = \sin(x)$



$$y = \frac{(\sin(0.85x) + \sin(0.9x) + \cdots + \sin(1.15x))}{7}$$

Given two waves of $y_1 = A\sin(kx - \omega t)$ and $y_2 = A\sin\big((k + dk)x - (\omega + d\omega)t\big)$, the superposition gives

$$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin\big((k + \Delta k)x - (\omega + \Delta\omega)t\big)$$

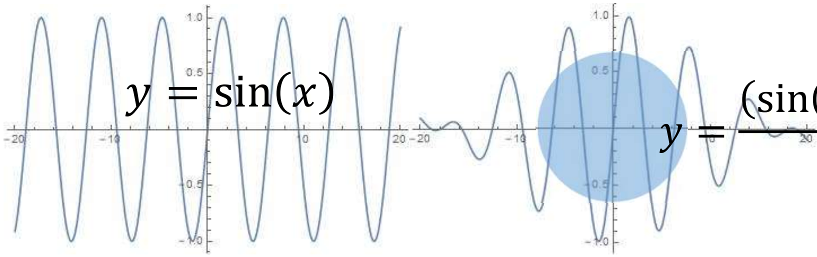
$$y = 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\sin\left(\left(k + \frac{\Delta k}{2}\right)x - \left(\omega + \frac{\Delta\omega}{2}\right)t\right)$$

$$\Delta k \ll k, \Delta\omega \ll \omega$$

$$y \cong 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\sin(kx - \omega t)$$

Ref:

Derive The Group Velocity



$y = \sin(x)$

$$y = \frac{(\sin(0.85x) + \sin(0.9x) + \cdots + \sin(1.15x))}{7}$$

$$y \cong 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\sin(kx - \omega t)$$

The wave speed, the phase velocity: $v_p = f\lambda = \dfrac{\omega}{k}$

The modulated amplitude $2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$ gives the group velocity (the speed of the wave packet).

The group velocity is: $v_g = \dfrac{\Delta\omega}{\Delta k} = \dfrac{d\omega}{dk} = \dfrac{d(\hbar\omega)}{d(k\hbar)} = \dfrac{dE}{dp}$

Compare it with the wave velocity of $v_p = \omega/k$.

Ref:

Velocity of Particles & EM Waves

	calculation
phase velocity	$f\lambda = \omega/k$
group velocity	$\Delta\omega/\Delta k = d\omega/dk$

The wave and the group velocity of **light** ($E = cp$) are

$$v_p = f\lambda = c, v_g = \frac{dE}{dp} = \frac{d(cp)}{dp} = c$$

The wave and the group velocity of a **particle** ($f = \frac{E}{h}, \lambda = \frac{h}{p}, E = \frac{p^2}{2m}$) are

$$v_p = f\lambda = \frac{E}{p} = \frac{p}{2m} = \frac{v}{2}, v_g = \frac{dE}{dp} = \frac{d(p^2/2m)}{dp} = \frac{2p}{2m} = v$$

The velocity of the particle is the group velocity.

	$E - p$ relation	v_p	v_g
particles	$E = p^2/2m$	$v/2$	v
photons	$E = cp$	c	c

EM Wave & de Broglie's Wave

EM Wave: $E(x,t) = E_0 \sin(kx - \omega t + \phi)$

$$B(x,t) = \frac{E_0}{c}\sin(kx - \omega t + \phi)$$

$$E = cp, E = hf, p = k\hbar$$

$$I = c\langle \varepsilon_{EM}\rangle = c\left\langle \frac{\epsilon_0}{2}E^2(x,t) + \frac{1}{2\mu_0}B^2(x,t)\right\rangle$$

$$I = c\langle \varepsilon_{EM}\rangle = c\langle \epsilon_0 E^2(x,t)\rangle = \frac{c\epsilon_0 E_0^2}{2}$$

Particle: $E = p^2/2m, f = E/h, \lambda = h/p$

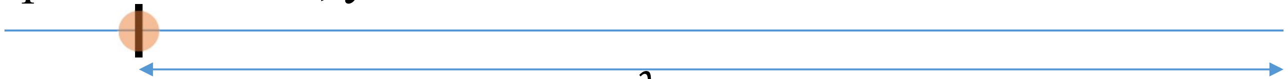
$$\Psi(x,t) = A_0 \sin(2\pi xp/h - 2\pi tE/h + \phi)$$

$$\Psi(x,t) = A_0 \sin(kx - \omega t + \phi)$$

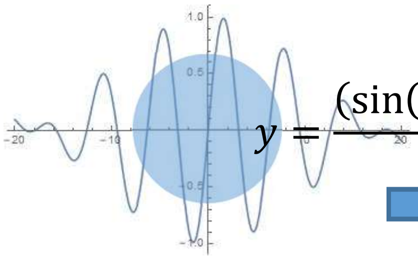
$$P(x) = \langle \Psi^2(x,t)\rangle_t$$

The Uncertainty Principle

particle nature, you cannot determine its λ and k



$\lambda \to \infty$



$$y = \frac{(\sin(0.85x) + \sin(0.9x) + \cdots + \sin(1.15x))}{7}$$

$\Rightarrow \Delta k \to \infty$

wave nature

finite λ $\Rightarrow \Delta x \to \infty$

The two natures are coexistent in EM waves and all particles.

What will the $\Delta x \Delta p$ be?

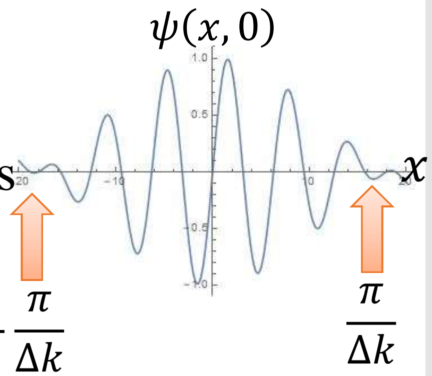
Constructing a wave for a particle

The Uncertainty Principle

$$\psi(x,t) = A\sin(kx-\omega t) + A\sin\big((k+\Delta k)x-\omega t\big)$$

$$\psi(x,t) \cong 2A\cos\left(\frac{\Delta k}{2}x\right)\sin(kx-\omega t)$$

wave packet Let $t=0$, $\psi(x,0)$ shows



The average position of the particle is at $x=0$.

The particle position spreads from $-\frac{\pi}{\Delta k}$ to $\frac{\pi}{\Delta k}$.

The standard position of the particle could be about a half of $\frac{\pi}{\Delta k}$.

$$\Delta x \cong \frac{\pi}{2\Delta k} \rightarrow \Delta x\Delta k \cong 1.57$$

This is not the smallest possible value for $\Delta x\Delta k$.

Transform between real space and periodic k space

Fourier Transform

$$f(x) = 1, 0 \le x \le L$$

$$f(x) = \frac{4}{\pi}\sin\left(\frac{\pi x}{L}\right) + \frac{4}{3\pi}\sin\left(\frac{3\pi x}{L}\right) + \frac{4}{5\pi}\sin\left(\frac{5\pi x}{L}\right) + \frac{4}{7\pi}\sin\left(\frac{7\pi x}{L}\right) + \cdots$$

$$f(x) = \sum_k A_k \sin(kx)$$

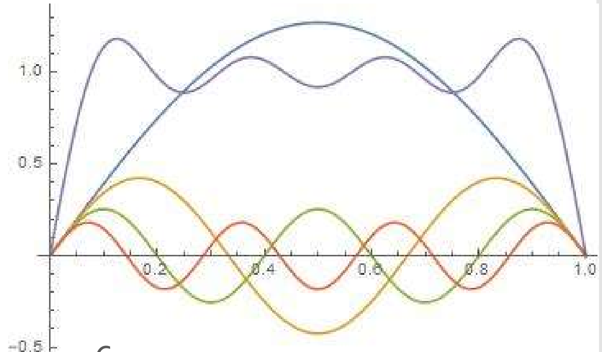
If $L \to \infty$, k is continuous & $A_k = A(k)$.

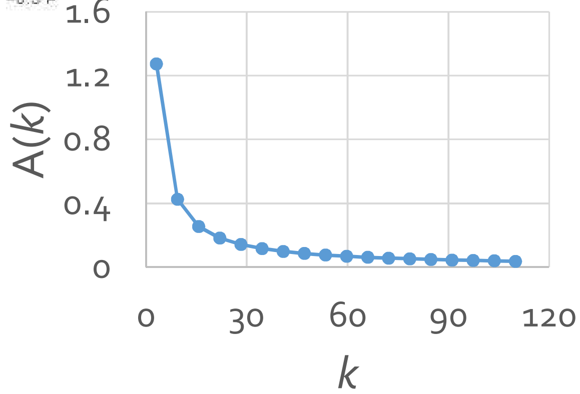
Extends to $-x$ and complex numbers:

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) e^{-ikx} dk$$

$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{ikx} dx$$

We obtain the Fourier transform to check the wave in either real space or k space.





The Optimum Uncertainty Value

The optimum uncertainty value is obtained for a Gaussian wave packet which is used to express a wave of a particle.

$$\psi(x,0) = \frac{A}{\sqrt{\sigma}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \exp(ikx)$$

The standard deviation of the Gaussian wave packet is $\Delta x = \frac{\sigma}{\sqrt{2}}$

The corresponding amplitude is obtained by the Fourier transform:

$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} \psi(x,0) dx$$

After we simply the equation, we have $A(k) = A\sqrt{\sigma} \exp\left(-\frac{k^2\sigma^2}{2}\right)$

The standard deviation of the amplitude in k space is $\Delta k = \frac{1}{\sqrt{2}\sigma}$

The optimum uncertainty value is $\Delta k \Delta x = \frac{1}{2}$, thus $\Delta k \Delta x \geq \frac{1}{2}$.

Other Uncertainty Equations

The uncertainty principle gives the standard deviation relation in both of the real space and the wave number k space as

$$\Delta x \Delta k \geq \frac{1}{2}$$

The momentum of the particle is $p = k\hbar$ thus $\Delta p = \hbar \Delta k$.

$$\Delta x \hbar \Delta k \geq \frac{1}{2}\hbar \rightarrow \Delta x \Delta p \geq \frac{\hbar}{2}$$

The energy-momentum relation gives $E = \frac{p^2}{2m} \rightarrow \Delta E = \frac{p}{m}\Delta p = v\Delta p$.

$$\Delta x \Delta p \geq \frac{\hbar}{2} \rightarrow \Delta x \frac{\Delta E}{v} \geq \frac{\hbar}{2} \rightarrow \frac{\Delta x}{v}\Delta E \geq \frac{\hbar}{2} \rightarrow \Delta t \Delta E \geq \frac{\hbar}{2}$$

The two forms of the uncertainty principle are:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

Examples

(a) Find the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 35°C. (b) Find the peak wavelength of the blackbody radiation emitted by the sun (5800 K).

$$\lambda_{max}T = 2.898 \times 10^{-3} \text{ m K}$$

(a) $T = 273.15 + 35 = 308.15 \text{ K}$

$$\rightarrow \lambda_{max} = \frac{2.898 \times 10^{-3}}{308.15} = 9.4 \times 10^{-6} \text{ (m)} = 9.4 \text{ μm}$$

(b) $T = 5800 \text{ K}$

$$\rightarrow \lambda_{max} = \frac{2.898 \times 10^{-3}}{5800} = 5.0 \times 10^{-7} \text{ (m)} = 500 \text{ nm}$$

Examples

A 3.0 kg block is attached to a massless spring which has a force constant k of 25 N/m. The spring is stretched and released at 0.50 m from its equilibrium. (a) Please find the total energy and the frequency of the oscillation from classical calculations. (b) Assume the energy of the oscillator is quantized, find the quantum number for the system oscillating with this amplitude.

(a) $m = 3.0, k = 25, A = 0.50$

$$E = \frac{1}{2}mv_m^2 = \frac{1}{2}kA^2 = \frac{1}{2}25 \times 0.50^2 = 3.13 \text{ J}$$

$$\omega = \sqrt{k/m} = \sqrt{25/3} = 2.89 \rightarrow f = \frac{\omega}{2\pi} = 0.459 \text{ Hz}$$

(b) $E_{quanta} = hf = 3.04 \times 10^{-34}$ J

$$E = 3.13 = N\left(3.04 \times 10^{-34}\right) \rightarrow N = 1.03 \times 10^{34}$$

Examples

Calculate the photon energies for light of wavelengths 400 nm (violet) and 700 nm (red).

$$E = hf = \frac{hc}{\lambda}$$

$$E_v = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{400 \times 10^{-9}} = 4.97 \times 10^{-19} J = 3.10 eV$$

$$E_r = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{700 \times 10^{-9}} = 2.84 \times 10^{-19} J = 1.77 eV$$

Assume that the intensity of sunlight is 1000 W/m^2 and that the average photon energy is 1.8 eV, calculate the number of photons that strike an area of 1 cm^2 each second.

$$E = P \times A \times \Delta t = 1000 \times (1 \times 10^{-4}) \times 1 = 0.1 \text{ J}$$

$$N = \frac{0.1}{1.8 \times 1.602 \times 10^{-1}} = 3.47 \times 10^{17}$$

Examples

The X-ray photon of wavelength 6 pm makes a head-on collision with an electron, so that the scattered photon goes in a direction opposite to that of the incident photon. The electron is initially at rest. (a) What is the wavelength of the scattered photon? (b) What is the kinetic energy of the recoiling electron?

(a)

$$\lambda_r - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta), \lambda_i = 6 \times 10^{-12}, \theta = \pi$$

$$\lambda_r = 6 \times 10^{-12} + \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}} \times 2 = 10.85 \times 10^{-12}$$

(b)

$$K = \Delta E = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_r} = hc\left(\frac{\lambda_r - \lambda_i}{\lambda_i \lambda_r}\right) = 1.48 \times 10^{-14}\ J$$

$$K = \frac{1.48 \times 10^{-14}}{1.602 \times 10^{-19}} = 9.24 \times 10^{4}\ \text{eV}$$

Examples

Find the de Broglie wavelength of a particle of mass 10^{-27} kg and a speed of 10^4 m/s.

$$p = mv = 10^{-27} \times 10^4 = 10^{-23}$$

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{10^{-23}} = 6.626 \times 10^{-11}\ \mathrm{m}$$

Find the de Broglie wavelength of an electron with mass m_e and kinetic energy K.

$$K = \frac{p^2}{2m} \rightarrow p = \sqrt{2mK}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{4.91 \times 10^{-19}}{\sqrt{K}}$$

If K is in the unit of eV, $\lambda = 1.23 \times 10^{-9}/\sqrt{K}$.